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Five Case Studies of Arithmetic Failures

Submitted by

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degree of Master of Education

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Statement

The present study is a minor research dealing with diagnostic and corrective procedures in the fundamentals of arithmetic. It takes the form of five case studies. Two of the children selected for study were referred by the Judge Baker Guidance Center; the other three by the principal of the Shurtleff School in Chelsea, Massachusetts.

Historical Sketch

The interest manifested in arithmetic becomes more understandable in the light of the historical development of the subject. Without such an introduction to the subject much of the research relating to it would appear meaningless, or at least isolated.

Arithmetic received little attention in the schools in the early period. In 1821, Warren Colburn prepared the first distinctly American text-book. He emphasized both the practical and disciplinary values of arithmetic, but he considered the practical values as more important. Soon the emphasis upon the practical values diminished, and by 1860 the chief objectives of arithmetic were the disciplinary ones. During the period of 1821 to 1900, the subject expanded too greatly. Some schools devoted as much as one-half of the total school time to this subject. By the end of the nineteenth century the practical influences were further diminished and text-books became only formal in character. (1)

About 1880 there appeared a public criticism aimed at the enormous amount of time devoted to the subject. (2) In 1893 and 1895, the Com-

(1) National Society for the Study of Education, 29th Yearbook, pp. 446-47

(2) Ibid, p. 447.

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mittee of Ten and the Committee of Fifteen urged reform in arithmetic. These committee reports pointed to the need of practical emphasis upon the subject rather than the disciplinary emphasis. (1)

Up to 1900 the developments in arithmetic were based chiefly upon empirical observations and a priori considerations. Prior to this year, very few objective data relating to the problem were available. In 1902 there appeared in print the published articles of Rice, which were based upon objective testing of the outcomes of arithmetic. Stone and Courtis began their testing in 1908 and 1909. Their results showed wide differences in the achievements of pupils. Following their findings several studies (2) soon appeared which dealt with the problem of elimination of subject matter. It became apparent that the earlier criticisms of over-expansion of arithmetic began to bear fruit. The two main types of investigations carried on were as follows: first, investigations consisted of an analysis of social demands, and second, investigations consisted of the psychological consideration of the number system and the difficulties which children meet in mastering it. (3)

The number of detailed summaries and of research studies now in print is extensive. At present the field of arithmetic is being very actively worked.

Results of Early Arithmetic Investigations

The course of study constructed for the Connersville, Indiana, school system, furnishes one of the earliest examples of objective application of elimination of useless topics from the subject. (4)

- (1) National Society for the Study of Education, 29th Yearbook, pp.447-48
- (2) Ibid, pp. 448-49
- (3) Ibid, p. 448
- (4) Wilson, Guy M. Connersville Course of Study in Elementary Mathematics, 1911. Republished 1916 by Warwick and York.

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Results of Early Arithmetic Investigations

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cation of elimination of useless topics from the subject. (4)

(1) National Society for the Study of Education, 25th Yearbook, pp. 447-448.

(2) Ibid., pp. 448-449.

(3) Ibid., p. 449.

(4) Wilson, G. M., Commercial Course of Study in Elementary Math-

In 1913 Courtis published the results of an investigation in New York City. The study was based upon the theory of social utility. It showed the differences in standards of achievement found in commercial activities in New York and in the New York public schools. Many such worthy investigations are reported, which are based upon the theory of social utility. Among the most extensive investigation based upon this theory are those reported by Wilson. (1)

Wilson (2) undertook an extensive investigation to determine the arithmetic used in both social and business situations. His study is most revealing. He found that much of the traditional material taught in the school could be eliminated on the basis of social and business usage.

In "What Arithmetic Shall We Teach?", Wilson (3) quotes the conclusions of a committee in charge of an investigation in Marshalltown, Iowa, on the construction of a course of study in arithmetic. The following are the conclusions:

1. The problems of arithmetic used by the public are simple. The necessary tool material could be taught in four years. We suggest grades 3, 4, 5, and 6. During these grades and after, motivated problems should lead the children out into actual business situations.
2. Much useless material is now in the curriculum in arithmetic. It should be eliminated. Only such material as will be useful in everyday life should be included.
3. The problems in arithmetic should center around business situations because life's problems in arithmetic do.
4. The community's arithmetic is sufficient to form the basis of the general arithmetic work in the elementary school.

(1) National Society for the Study of Education, 29th Yearbook, p. 449.
 (2) Wilson, Guy M. What Arithmetic Shall We Teach?
 (3) Ibid, pp. 58-59.

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Judd (1) reports that everywhere in the United States school systems are engaged in reorganizing the curriculum. The policy of social utility will do much toward a simplification of the arithmetic curriculum. This needed change is consistent with the social needs, and, undoubtedly, will bring about higher standards in performance.

According to surveys of instruction a major cause of non promotion in the elementary grades is failure in the subject of arithmetic. More pupils have failed in this subject than in any other subject of the curriculum. (2)

A study by Edwards (3) relating to pupils knowledge of the fundamental processes illustrates the lack of mastery of the fundamental processes on the part of elementary school graduates. In Racine, Wisconsin, a test was administered to pupils from the fifth to the twelfth grade gave equally discouraging results. (4)

While there are much data on pupil and class scores on tests, there is still insufficient information concerning the causes of such scores. Much more scientific information is needed. In recent years, important studies have been concerned with the nature of the psychological difficulties of the children who are making poor progress in arithmetic. These studies have given significant information as to the most common types of errors made by children, their habits of work, the faulty procedures they use, the nature of the difficulties they encounter in problem solving, their general physical and mental characteristics, and their social qualities.

-
- (1) Judd, Charles H. Psychological Analysis of the Fundamentals of Arithmetic, p. 116.
 - (2) Brueckner, Leo J. Diagnostic and Remedial Teaching in Arithmetic, p. 1.
 - (3) Edwards, William H. Bridging the Gap between Theory and Practice in Ninth-grade Mathematics, School Science and Mathematics, XXVIII (November, 1928), pp. 846-66.
 - (4) Potter, Mary. An Attempt to Improve Computation, Mathematics Teacher, XX (November, 1927), pp. 381-85.

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- (1) Lutz, Charles H. Psychological Analysis of the Fundamentals of Arithmetic, p. 110.
- (2) Bracchieri, Leo J. Diagnostic and Remedial Teaching in Arithmetic, p. 110.
- (3) Edwards, William A. Bridging the Gap between Theory and Practice in Arithmetic, p. 110.
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Stone (2) in his earlier studies reported that arithmetic is made up of a large number of specific abilities each of which must be developed through careful practice. The results of this and similar studies have revealed some of the specific abilities and skills which the children need to have in order to master the fundamental processes of arithmetic.

The attempts to improve instructional materials have naturally followed the revelations on the nature of pupil errors. Some of this has been well worked out; some has merely increased the burden for the child.

In the present study, Wilson drill books have been used as a basis for corrective work. These books are based upon; first, a careful analysis of the facts in a process, and, second, the process steps which are required by pupils' actual difficulties. The result is a teaching plan very simple and direct. A basic assumption is that work will be deferred until meaning and motivation have been built up in the children.

Studies concerned with the diagnosis of pupil difficulties have made it clear that many factors contribute to the normal progress of pupils in mastering arithmetic processes. Such studies have influenced the teaching procedures in the classroom. They have revealed some of the methods employed by children in working with the fundamental processes of arithmetic.

- (1) National Society for the Study of Education, 29th Yearbook, p. 451.
- (2) Stone, C.W. Arithmetic Abilities and Some Factors Determining Them, Teachers College Contributions to Education, No. 19.

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(1) National Society for the Study of Education, 25th Yearbook, p. 431.
(2) Stone, C.W. Arithmetic Abilities and Some Factors Determining Them, Teachers College Contributions to Education, No. 19.

An unpublished Master's thesis by Smith (1) is one of the earliest examples of diagnostic and remedial work in arithmetic. In 1917, two years later, Uhl (2) made one of the most significant contributions to the technique of diagnosis in arithmetic. In 1923 Brueckner (3) reported an excellent plan for diagnosis in arithmetic.

Recently a diagnostic and remedial experiment was conducted with the cooperation of some seventy teachers. In this experiment (4) the individual pupil's oral and written responses were analyzed in order to discover the methods of work employed by these children. Specific remedial measures were also applied.

On the findings of these and similar studies it has been possible to arrive at a fairly definite statement as to the kinds of faulty, peculiar methods of procedure that are found in the work of pupils who are not making satisfactory progress. Educational literature contains many interesting descriptions of the peculiar methods of work that have been discovered.

Some factors which contribute to the failure of mastering the arithmetic processes are of a permanent kind; others are temporary and may be obviated. These contributing factors vary widely from pupil to pupil and may appear in various combinations of the following:

1. Lack of mentality or native ability
2. Physical handicaps
3. Emotional factors
4. Faulty attitudes of the pupil toward his school work

(1) Buswell, Guy T., and Judd, Charles H. Summary of Educational Investigations Relating to Arithmetic, p. 115.

(2) Ibid, pp. 117-18.

(3) Ibid, pp. 123-27.

(4) Buswell, Guy T., and John, Lenore. Diagnostic Studies in Arithmetic.

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(1) Brueschman, Guy T., and Lewis, Charles H. Summary of Educational Investigations Relating to Arithmetic, p. 115.
 (2) Uhl, p. 112-13.
 (3) Uhl, p. 123-24.
 (4) Brueschman, Guy T., and Lewis, Charles H. Diagnostic Studies in Arithmetic.

5. Pedagogical factors over which the school has some control (1) These may be traceable to such contributing causes as home influences, methods of teaching, habits of study, content of instruction, and specific difficulties in the subject.

In order to make a diagnosis of the causes of a pupil's deficiency in arithmetic the teacher should have a grasp of the skills that constitute the processes, and an appreciation of how complex the processes must appear to the learner, also a knowledge of the most common causes and kinds of difficulties revealed in the study of the work of the pupils in the grades. (2) Such information should be supplemented by a knowledge of:

1. The child's behavior
2. The child's school records
3. The results of diagnostic tests
4. Conferences with the classroom teachers
5. The results of informal tests

It is not enough to know that a certain pupil is failing or is deficient in the fundamentals. It is necessary to know what errors are made and if possible to determine the causes of these errors. They may be due to such factors as; improper habits of work, lack of attention, inadequate or incorrect first learning of arithmetic facts, insufficient drill, or any other or a combination of the many causes which have been found to exist when individual pupil difficulties have been studied. Carelessness is too often assigned as a cause; it is better to consider it as a symptom and go behind it for causes.

- (1) Brueckner, Leo J. Diagnosing Pupil Difficulties, Journal of the National Education Association, 21:123-25, April, 1932.
- (2) Brueckner, Leo J. Diagnostic and Remedial Teaching in Arithmetic, p. 110.

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Carelessness is too often assumed as a cause; it is better to consider

it as a symptom and go behind it for causes.

In an attempt to discover errors and their possible causes, Lazar (1) used the following methods in her study:

1. Individual record sheet
2. Analysis of pupils' test paper
3. Observation of the pupils as they worked
4. Individual oral examination
5. Use of inventory tests

Test scores aid the teacher to locate possible places where difficulty exists, but they do not locate the causes of difficulty. In testing the teacher should keep a record of the time taken to complete the test, but this should not be considered as the most important factor. However, it helps the teacher to locate in some cases poor methods of work and lack of mastery of the processes. The time element in such cases will greatly assist in diagnosis. It is the opening key to an analysis of the child's methods of work, the writer has found.

Before undertaking a remedial program such factors as physical handicaps, emotional factors, and environmental factors should be corrected.

Dr. Brueckner (2) states that there appears to be at least three essential elements in corrective work:

- (1) Developing a purposeful cooperative attitude of the pupil; (2) correcting minor deficiencies due to temporary difficulties or specific gaps in training; and (3) general reteaching in cases which exhibit such serious deficiencies that a restudy of essential fundamentals is necessary.

Dr. Brueckner further states that:

The necessity for remedial teaching in the case of any pupil indicates the presence of an emergency situation which must be

- (1) Lazar, May. Diagnostic and Remedial Work in the Arithmetic Fundamentals, p. 30.
- (2) Brueckner, Leo J. Remedial Cases, Journal of the National Education Association, 21:147-48, May, 1932.

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- ulation is necessary.

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The necessity for remedial teaching in the case of any pupil

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(1) Jager, Mary. Diagnostic and Remedial Work in the Arithmetic Room--
Boston, p. 75.
(2) Brincker, Mrs. J. L. Remedial Cases, Journal of the National Association
of Teachers, 21:17-22, May, 1925.

characterized by a loss of interest, disinclination to exert himself, and indifference as to the outcomes of his training. Exhortations to effort, threats of punishment, promises of rewards, and similar devices have little effect. In such cases it is vital that new interest be created thru helpful instruction by removing the conditions that cause failure and building up in the pupil the feeling of satisfaction that results from successful efforts. This may be accomplished by pointing out efficient methods of attack, using simpler types of material, well graded practice exercises, progress charts, and a variety of attractive, interesting activities showing the utility of number.

In the study reported herein the writer has kept in mind some such background of Educational History as has been given in this sketch. The assumption has been that the faults that these children have exhibited can be corrected; and that in useful arithmetic success and perfect scores are possible.

43 This test contains 55 primary multiplications that require borrowing.

44 This test is the multiplication process test. The long form contains 100 examples. The short form or revised form contains 50 examples. The first four examples are in five parts each.

Multiplication

45 This test contains 100 multiplications to 9 x 9.

46 This is the multiplication process test. The long form contains 100 examples. The short form or revised form contains 50 examples. The first four examples are in five parts each.

Division

47 This test contains 51 even quotients to 9's in 51.

48 This is the division process test. It contains 50 examples in long division.

characterized by a loss of interest, dissatisfaction, and indifference as to the outcome of his training. Experiments to effort, threats of punishment, promises of rewards, and similar devices have little effect. In such cases it is vital that new interest be created first before any further instruction by removing the conditions that cause failure and building up in the pupil the feeling of satisfaction that results from successful effort. This may be accomplished by pointing out efficient methods of attack, using simpler types of material, well graded practice exercises, progress charts, and a variety of attractive, interesting activities showing the ability of man.

In the study reported herein the writer has kept in mind some such

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Description of Tests

Key Addition

- 3A This test contains 50 easy primary combinations
- 3B This test contains 50 difficult combinations.
- 3C This test contains 500 decade combinations to $39 + 9$.
- 3P This test is the addition process test. The long form contains 100 examples. The short form or revised form contains 25 examples. No. 1 is in ten parts.

Subtraction

- 4A This test contains 55 primary combinations requiring no borrowing.
- 4B This test contains 45 primary combinations that require borrowing.
- 4P This test is the subtraction process test. The long form contains 100 examples. The short form or revised form contains 25 examples. The first four examples are in five parts each.

Multiplication

- 5A This test contains the 100 combinations to 9×9 .
- 5P This is the multiplication process test. The long form contains 100 examples. The short form or revised form contains 25 examples. Part a and e have ten parts each.

Division

- 6A This test contains 81 even quotients to 9's in 81.
- 6P₂ This is the division process test. It contains 50 examples in long division.

Description of Tests

Addition

207

This test contains 30 easy primary combinations.

21

This test contains 30 difficult combinations.

22

This test contains 300 decimal combinations to 32 + 2.

23

This test is the addition process test. The long form contains 100 examples. The short form or revised form contains 25 examples. No. 1 is in ten parts.

24

Subtraction

This test contains 30 primary combinations regarding no borrowing.

25

This test contains 40 primary combinations that require borrowing.

26

This test is the subtraction process test. The long form contains 100 examples. The short form or revised form contains 25 examples. The first four examples are in five parts each.

27

Multiplication

This test contains the 100 combinations to 2 x 2.

28

This is the multiplication process test. The long form contains 100 examples. The short form or revised form contains 25 examples. Part 2 and 3 have ten parts each.

29

Division

This test contains 81 even dividends to 2's in 81.

30

This is the division process test. It contains 30 examples in long division.

31

CASE I

Roland was attending a sight-saving class in Newton at the time this study was made. He was born November 21, 1921. His intelligence quotient was found to be 85 according to the Otis Group Intelligence Test. (1)

He was referred for study by the Judge Baker Guidance Center because he was deficient in arithmetic.

School History. (2) Roland has had many shifts at school. He entered school at New Britain, Connecticut. Later he attended a French orphanage, and a public school in Lowell, Massachusetts.

In March, 1934 he was referred to the Lowell Habit Clinic by the school authorities because of truancy. At school, whenever he was late or tardy, he offered all sorts of excuses. In his academic work he could not concentrate. The school authorities considered him mentally deficient and were surprised to learn that he was a normal boy.

It was in November, 1934 that he was referred to the Judge Baker Guidance Center. According to their findings he was retarded two years in school grade. They found that his comprehension in reading was well up to grade, and his spelling almost up to grade, but his arithmetic very poor.

At present Roland is repeating the 6th grade.

Personality of Child. His general appearance is marred somewhat by thick glasses which he has to wear because of very defective vision. He is decidedly aggressive and talkative, and shows an excellent persistence and effort. He has some leadership qualities and likes to be active.

He is very much attached to his mother. When he was at home he was

(1) Case study No. 9163. Judge Baker Guidance Center

(2) Ibid.

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disobedient. He was described as excitable, and often engaged in defense lying. He would not help around the home unless bribed or threatened.

Home Conditions. (1) Roland is the oldest of four children. He has two sisters, one eight years old, the other six years old. The youngest member of the family is a boy four years old.

The mother is fairly intelligent and much interested in the children. She completed the eighth grade at fourteen.

The father's mental age was found to be 13.7 years. In 1932 he was diagnosed as definitely neurotic. He did not complete grammar school. He worked as a steeple-jack, and his work kept him away from home a great deal. He never contributed to the childrens' support, consequently the family had to depend upon Mothers Aid.

Many complications arose in the family and finally the mother received a divorce from the father, which became final in October, 1930.

Roland is now in the care of the Boston Children's Aid. This organization placed him in a foster home in which he lived in during the time this study was made.

Results of Tests in Arithmetic. Table I gives the initial results of the Wilson Inventory and Diagnostic Tests. The time taken to complete each test was not recorded in every case. The reason for that is, that there was some question about the accuracy of the time in such cases. In cases where time is recorded it reads in minutes and seconds, for example, 4:00 is read 4 minutes and 0 seconds.

(1) Ibid.

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TABLE I

**Initial Results of Case 1 on the Wilson Inventory
and Diagnostic Tests**

Process	Date	Test	Score	Time
Addition	1/29/35	3A	100
	1/29/35	3B	99
	1/29/35	3C	96+
	1/29/35	3P (short form)	84
Subtraction	1/31/35	4A	98	4:00
	1/31/35	4B	99	10:00
	1/31/35	4P (long form)	93	39:30
Multiplication	2/5/35	5A	95	8:00
	2/5/35	5P (long form)	78	44:20
Division	2/7/35	6A	98	7:00
	2/7/35	6P ₂	* ..	31:00

* Score not recorded because the entire test was not completed. The child attempted 35 examples out of a possible 50, only 15 were correct. The remaining 15 were too difficult for him to do.

TABLE I
Initial Results of Case I on the Wilson Inventory
and Diamond's Tests

Process	Date	Test	Score	Time
Addition	1/29/33	3A	100
	1/29/33	3B	99
	1/29/33	3C	98
	1/29/33	3D (short form)	94
Subtraction	1/31/33	4A	98	4:00
	1/31/33	4B	99	10:00
	1/31/33	4C (long form)	95	29:30
Multiplication	2/5/33	5A	95	8:00
	2/5/33	5B (long form)	78	44:30
Division	2/7/33	6A	98	7:00
	2/7/33	6B	..	31:00

* Score not recorded because the entire test was not completed. The child attempted 25 examples out of a possible 30, only 12 were correct. The remaining 13 were too difficult for him to do.

Analysis of Difficulties. A careful diagnosis was made of Roland's habits of work in dealing with the four fundamental processes. The diagnosis was based upon the results of the various tests. The following tabulation shows the errors and the inefficient habits of work:

Addition

1. Error in primary combination
2. Errors in upper decade combinations

Subtraction

1. Errors in primary combinations
2. Last subtraction a zero brought down
3. Errors due to a succession of zeros in minuend
4. Deducted two from minuend after borrowing

Multiplication

1. Errors in primary combinations
2. Misplaced decimal point (U.S. Money)
3. Left out dollar sign (U.S. Money)
4. Error in adding carried number
5. Errors in addition
6. Forgot to add carried number

Division

1. Errors in division combinations
2. Used remainder larger than divisor
3. Errors in subtraction
4. Errors in multiplication
5. Disregarded digit in dividend
6. Disregarded a column within the example

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Addition

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2. Errors in upper decade combinations

Subtraction

1. Errors in primary combinations
2. Last subtraction a zero brought down
3. Errors due to a succession of zeros in dividend
4. Borrowed two from dividend after borrowing

Multiplication

1. Errors in primary combinations
2. Missplaced decimal point (U.S. Money)
3. Left out dollar sign (U.S. Money)
4. Error in adding carried number
5. Errors in addition
6. Forgot to add carried number

Division

1. Errors in division combinations
2. Used remainder larger than divisor
3. Errors in subtraction
4. Errors in multiplication
5. Misplaced digit in dividend
6. Misplaced a column within the example

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests are illustrated below. They were taken from the various tests and appear in the exact form.

Difficulties in Addition

1. Error in primary combination

Only one primary combination was missed, the following:

$$\begin{array}{r} 6 \\ 0 \\ \hline 0 \end{array}$$

2. Errors in upper decade combinations

The following upper decade combinations were missed:

$$23 + 2 = 5 \qquad 20 + 5 = 5$$

$$38 + 7 = 46 \qquad 35 + 7 = 45$$

$$27 + 7 = 35 \qquad 35 + 9 = 42$$

$$39 + 5 = 43 \qquad 28 + 5 = 32$$

$$32 + 6 = 8 \qquad 33 + 9 = 41$$

Difficulties in Subtraction

1. Errors in primary combinations

The following primary combinations were missed:

$$\begin{array}{r} 8 \\ 0 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 3 \\ 2 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 11 \\ 4 \\ \hline 9 \end{array}$$

2. Last subtraction a zero brought down

Example 1:

$$\begin{array}{r} 96 \\ 91 \\ \hline 05 \end{array}$$

In this example the last subtraction a zero was brought down.

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests are illustrated below. They were taken from the various tests and appear in the exact form.

Difficulties in Addition

1. Error in primary combination
Only one primary combination was missed, the following:

$$\begin{array}{r} 3 \\ 0 \\ \hline 7 \end{array}$$

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The following upper decade combinations were missed:

$23 + 2 = 5$	$23 + 2 = 5$
$23 + 7 = 43$	$23 + 7 = 43$
$23 + 9 = 43$	$23 + 9 = 43$
$23 + 3 = 32$	$23 + 3 = 42$
$23 + 2 = 41$	$23 + 2 = 8$

Difficulties in Subtraction

1. Error in primary combinations
The following primary combinations were missed:

$\begin{array}{r} 11 \\ 4 \\ \hline 7 \end{array}$	$\begin{array}{r} 2 \\ 3 \\ \hline 5 \end{array}$	$\begin{array}{r} 8 \\ 0 \\ \hline 7 \end{array}$
----------------------------------------------------	---------------------------------------------------	---------------------------------------------------

2. Last subtraction a zero brought down

Example 1:

In this example the last subtraction a zero was brought down.

$$\begin{array}{r} 96 \\ 91 \\ \hline 05 \end{array}$$

Example 2:

$$\begin{array}{r} 71 \\ 62 \\ \hline 09 \end{array}$$

The last subtraction in this example is a zero, it was brought down.

3. Errors due to a succession of zeros in minuend

Example 1:

$$\begin{array}{r} 1200 \\ 940 \\ \hline 360 \end{array}$$

This boy worked the example on the left as follows: "0 from 0 leaves 0; 4 from 10 leaves 6; 9 from 12 leaves 3."

Example 2:

$$\begin{array}{r} \$15.00 \\ 1.50 \\ \hline \$14.50 \end{array}$$

He worked this example as follows: "0 from 0 leaves 0; 5 from 10 leaves 5; 1 from 5 leaves 4 and bring down the 1."

4. Deducted two from the minuend after borrowing

Example 1:

$$\begin{array}{r} \$10.00 \\ 5.98 \\ \hline \$ 3.02 \end{array}$$

In this example the boy deducted 2 instead of 1 from the 10 in the last subtraction. He apparently said, 5 from 8 leaves 3.

Example 2:

$$\begin{array}{r} \$10.00 \\ 6.25 \\ \hline \$ 2.75 \end{array}$$

In this example, it seems, he followed exactly the same procedure in subtracting as he did in the example directly above.

Difficulties in Multiplication

1. Errors in primary combinations

The following combinations were missed:

$\frac{3}{2}$	$\frac{2}{5}$	$\frac{7}{3}$	$\frac{6}{24}$	$\frac{7}{7}$
---------------	---------------	---------------	----------------	---------------

Example 2:

71
38
09
The last subtraction in this example is a zero,
it was brought down.

3. Errors due to a succession of zeros in minuend

Example 1:

1200
240
090
This boy worked the example on the left as follows:
"0 from 0 leaves 0; 4 from 10 leaves 6; 2 from 12
leaves 2."

Example 2:

\$12.00
1.50
\$14.50
He worked this example as follows: "0 from 0 leaves
0; 5 from 10 leaves 5; 1 from 2 leaves 1 and bring
down the 1."

4. Deducted two from the minuend after borrowing

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2.38
\$ 7.62
In this example the boy deducted 2 instead of 1
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Example 2:

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4.38
\$ 5.62
In this example, it seems, he followed exactly the
same procedure in subtracting as he did in the ex-
ample directly above.

Difficulties in Multiplication

1. Errors in primary combinations

The following combinations were missed:

3	2	7	6	7
2	2	6	2	0
2	2	24	14	7

2. Misplaced decimal point (U.S. Money)

Example 1:

$$\begin{array}{r} \$.27 \\ 100 \\ \hline \$.2700 \end{array}$$

In this example he misplaced the decimal point.

Example 2:

$$\begin{array}{r} \$5.90 \\ 10 \\ \hline \$5.900 \end{array}$$

Another illustration of the same difficulty.

Example 3:

$$\begin{array}{r} \$16.80 \\ 300 \\ \hline \$50.4000 \end{array}$$

In this example the multiplication is correct, but the decimal point is misplaced.

3. Left out the dollar sign (U.S. Money)

Example:

$$\begin{array}{r} \$7.50 \\ 6 \\ \hline 45.00 \end{array}$$

In the example on the left, he left out the dollar sign.

4. Error in adding the carried number

Example:

$$\begin{array}{r} 78 \\ 8 \\ \hline 604 \end{array}$$

In this example, he failed to add correctly the carried number from the first multiplication.

5. Errors in addition

Example 1:

$$\begin{array}{r} 54 \\ 270 \\ \hline 3780 \\ 108 \\ \hline 13780 \\ * \end{array}$$

In adding the partial products in this example, he not only added incorrectly, (position indicated by asterisk) but also failed to add the carried number in the following column.

Example 2:

$$\begin{array}{r}
 923 \\
 21 \\
 \hline
 923 \\
 1846 \\
 \hline
 21683 \\

 \end{array}$$

In this example, he added three columns incorrectly. The positions are indicated by asterisks. It seems as though he may have multiplied the numbers in the third column, and from there on proceeded to add again.

6. Forgot to add carried number

Example:

$$\begin{array}{r}
 \$680 \\
 120 \\
 \hline
 12600 \\
 680 \\
 \hline
 80600
 \end{array}$$

In this example, he failed to add the carried number. He multiplied by 2 as follows: "2 times 0 is 0; 2 times 8 is 16; 2 times 6 is 12."

Difficulties in Division

1. Errors in division combinations

The following were missed:

$$1) \frac{1}{9} \qquad 5) \frac{5}{5}$$

2. Used remainder larger than divisor

Example 1:

$$\begin{array}{r}
 26 \text{ R-11} \\
 73) \overline{2409} \\
 \underline{146} \\
 *949 \\
 \underline{438} \\
 11
 \end{array}$$

In working this example, he used a remainder larger than the divisor. The place is indicated by the asterisk.

Example 2:

$$\begin{array}{r}
 4451 \text{ R-46} \\
 31) \overline{139996} \\
 \underline{124} \\
 159 \\
 \underline{124} \\
 *359 \\
 \underline{155} \\
 *46
 \end{array}$$

In working this example, he had a remainder in two cases, which was larger than the divisor. The asterisks indicate the positions.

Example 2:

$$\begin{array}{r} 123 \\ 45 \\ \hline 578 \\ 123 \\ \hline 701 \end{array}$$

In this example, he added three columns incorrectly. The positions are indicated by asterisks. It seems as though he may have carried the numbers in the third column, and from there on proceeded to add again.

6. Forgot to add carried number

Example:

$$\begin{array}{r} 4000 \\ 120 \\ \hline 1200 \\ 200 \\ \hline 6000 \end{array}$$

In this example, he failed to add the carried number. He multiplied by 2 as follows: "2 times 0 is 0; 2 times 0 is 0; 2 times 0 is 0."

Difficulties in Division

1. Errors in division combinations

The following were missed:

$$\begin{array}{r} 1 \\ 1 \overline{) 2} \\ \hline 2 \end{array}$$

2. Used remainder larger than divisor

Example 1:

$$\begin{array}{r} 26 \\ 2 \overline{) 54} \\ \hline 10 \\ 10 \\ \hline 44 \\ 44 \\ \hline 0 \end{array}$$

In working this example, he used a remainder larger than the divisor. The place is indicated by the asterisk.

Example 2:

$$\begin{array}{r} 44518-16 \\ 21 \overline{) 13978} \\ \hline 42 \\ 139 \\ \hline 139 \\ 139 \\ \hline 0 \end{array}$$

In working this example, he had a remainder in two cases, which was larger than the divisor. The asterisks indicate the positions.

3. Errors in subtraction

Example:

$$\begin{array}{r}
 98 \text{ R-44} \\
 47 \overline{) 4300} \\
 \quad *376 \\
 \quad \underline{440} \\
 \quad \quad 376 \\
 \quad \quad \underline{44}
 \end{array}$$

In subtracting at the point indicated by the asterisk, he said, "6 from 10 leaves 4; 7 from 12 leaves 4." He also multiplied incorrectly by 9.

4. Errors in multiplication

Example:

$$\begin{array}{r}
 53 \text{ R-63} \\
 64 \overline{) 3445} \\
 \quad \underline{320} \\
 \quad \quad 245 \\
 \quad \quad \underline{182} \\
 \quad \quad \quad 63
 \end{array}$$

He multiplied 3 by 64 and obtained 182 as the product.

5. Disregarded digit in dividend

Example:

$$\begin{array}{r}
 71 \\
 26 \overline{) 18460} \\
 \quad \underline{182} \\
 \quad \quad 26 \\
 \quad \quad \underline{26} \\
 \quad \quad \quad 000
 \end{array}$$

He worked as shown, failing to make use of the last 0 in the dividend.

6. Disregarded a column within the example

Example:

$$\begin{array}{r}
 44 \text{ R-44} \\
 81 \overline{) 5508} \\
 \quad \underline{324} \\
 \quad \quad 368 \\
 \quad \quad \underline{324} \\
 \quad \quad \quad 44
 \end{array}$$

In working this example, he left out the last column in the subtraction after the first multiplication.

Errors in subtraction

Example:

$$\begin{array}{r} 984 \\ 47 \times 200 \\ \hline 9680 \\ 9840 \\ \hline 19520 \end{array}$$

In subtracting at the point indicated by the asterisk, he said, "8 from 10 leaves 2; 7 from 10 leaves 3." He also multiplied incorrectly by 9.

Errors in multiplication

Example:

$$\begin{array}{r} 63 \\ 34 \times 3 \\ \hline 102 \\ 132 \\ \hline 198 \end{array}$$

He multiplied 3 by 34 and obtained 102 as the product.

Disregarded digit in dividend

Example:

$$\begin{array}{r} 71 \\ 25 \times 1848 \\ \hline 2000 \\ 2000 \\ \hline 4000 \end{array}$$

He worked as shown, failing to make use of the last 0 in the dividend.

Disregarded a column within the example

Example:

$$\begin{array}{r} 44 \\ 81 \times 308 \\ \hline 2448 \\ 2448 \\ \hline 4896 \end{array}$$

In working this example, he left out the last column in the subtraction after the first multiplication.

General Plan of Remedial Instruction. The general plan of corrective instruction was to remove the faulty habits and to strengthen the weaker habits in the four fundamental processes, which were revealed by the testing program. The attack was concerned with better presentation of the processes and facts, and further drill on the processes and facts after presentation.

The diagnosis revealed that the boy had special handicaps in each of the four fundamental operations. The processes were attacked in the following order: first, addition; second, subtraction; third, multiplication; and fourth, division. The Wilson 100% Drill Services were used to supply the practice exercises in order to strengthen the good habits once established. The exercises in these drill services are well graded, and proceed step by step in terms of gradually increasing difficulty.

During the corrective program the various difficulties were treated as they arose or as they were introduced in the drill services. Such a procedure was followed throughout the entire remedial program, but occasionally it was necessary to deviate from this plan, because errors appeared which were to be treated later in the program. In other words a fault was never allowed to pass unnoted, it was immediately corrected, explained, and illustrated by a better method.

Always the child was praised for doing good work, and for overcoming faulty habits. From time to time informal examinations were administered to check for mastery and to discover whether or not undesirable habits fell into disuse. During the instructional periods his work was carefully watched, and his methods of thinking through examples were checked by having him work the examples aloud.

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Detailed Account of Remedial Instruction. The methods employed to improve the child's habits of work are explained more in detail in the following paragraphs. The corrective measures used to correct various specific difficulties are described. Reference to the list of the detailed description of habits given in the early part of this Case will enable the reader to identify the habits being considered at any time.

Addition

Flash cards were constructed and used to overcome the errors in the primary combinations in each of the four fundamental processes. Also further drill was given in the other primary facts to insure both speed and accuracy. Whenever he hesitated too long on a combination, it was set aside for further study. The combinations to which he responded quickly enough, were set aside, also, to keep the two groups separate. This plan was employed until he mastered the combinations.

After having mastered the primary facts of a group, he received practice on the related facts of that group. He studied these facts from individual flash cards.

Subtraction

To overcome subtraction habit No. 2, he was just told that the zero in the last subtraction need not be brought down. Then, it was also explained that the zero in the answer does not make the answer incorrect, but that the accepted practice is to leave out the zero in such cases. Practice exercises followed, and from time to time similar examples were given to check if the habit fell into disuse.

Subtraction habit No. 3 was corrected by illustrating the method of

23
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Subtraction

To overcome subtraction habit No. 2, he was first told that the zero in the last subtraction need not be brought down. Then, it was explained that the zero in the answer does not make the answer incorrect, but that the accepted practice is to leave out the zero in such cases. Practice exercises followed, and from time to time similar examples were given to check if the habit fell into place. Subtraction habit No. 3 was corrected by illustrating the method of

of crossing out the number whenever one borrows from it, and to write directly above it the number that is left. He accepted this plan without any questions. The result was that a succession of zeros in the minuend no longer introduced difficulties.

Subtraction habit No. 4 appeared in the testing program, however, it did not show up in the instructional periods. Perhaps it was corrected by having the child cross out whenever he borrowed. This method is explained in the paragraph above.

Multiplication

To overcome multiplication habits 2 and 3, the meaning and purpose of the decimal point and the dollar sign was explained. Following this, practice exercises were given to break down these faulty habits. From time to time examples in the drill services were used to check for mastery.

Whenever multiplication habits 4, 5, and 6 appeared during the instructional periods, Roland's attention was immediately called to them. He was requested to do the same example over again, and usually he would obtain the correct answer. Apparently some factor was present which brought this about. Then, he was requested to compare the two answers, and discover for himself the error that was made in the first example. Sometimes he was able to discover the error himself; other times it was necessary to point it out and explain it to him.

Division

To overcome the habit of using a remainder larger than the divisor; first, the mistake was pointed out to him; and second, the correct method of doing long division examples was illustrated. Following this, practice

of crossing out the number whenever one borrows from it, and to write directly above it the number that is left. He accepted this plan without any questions. The result was just a succession of zeros in the mind and no longer introduced difficulties.

Subtraction Habit No. 4 appeared in the testing program, however, it did not show up in the instruction periods. Perhaps it was corrected by having the child cross out whenever he borrowed. This method is explained in the paragraph above.

Multiplication

To overcome multiplication habits 1 and 2, the morning and purpose of the decimal point and the dollar sign was explained. Following this, practice exercises were given to break down these faulty habits. From time to time examples in the drill services were used to check for mastery. Whenever multiplication habits 4, 5, and 6 appeared during the instructional periods, Roland's attention was immediately called to them. He was requested to do the same example over again, and usually he would obtain the correct answer. Apparently some factor was present which brought this about. Then, he was requested to compare the two answers, and discover for himself the error that was made in the first example. Sometimes he was able to discover the error himself; other times it was necessary to point it out and explain it to him.

Division

To overcome the habit of using a remainder larger than the divisor, first, the mistake was pointed out to him; and second, the correct method of doing long division examples was illustrated. Following this, practice

exercises were assigned to insure mastery.

Whenever division habits 3 and 4 appeared, he was requested to do that part of the example aloud, and make the correction.

To overcome division habit No. 5, the correct method of doing similar examples was illustrated. Then, he was requested to do several such examples for practice.

Division habit No. 6 appeared in the testing program, but it did not appear in the instructional periods. Perhaps it may have been due to illegible figures or cramped conditions on the test. Instead of erasing numbers he would write over them. During the instructional periods he was encouraged to erase always so that the figures might be clearer. The size of the figures was large, but readable.

Final Test Results. At the conclusion of the remedial instruction the Wilson Inventory and Diagnostic Tests were administered in each of the fundamental processes. The results of the final tests are listed in Table II on the following page. Tests 3C and 6P₂ were given in two and three installments respectively. In such cases the date of the first installment appears, the total score, and the total time taken to complete the entire test.

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Table II on the following page. Tests 30 and 31 were given in two and

three installments respectively. In each case the date of the first

installment appears, the total score, and the total time taken to com-

plete the entire test.

TABLE II

Final Results of Case 1 on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	2/ 2/35	3A	100	3:40
	2/ 2/35	3B	100	5:40
	2/14/35	3C	98+	17:45
	2/14/35	3P (short form)	96	9:30
Subtraction	3/ 5/35	4A	100	3:55
	3/ 5/35	4B	100	5:30
	3/15/35	4P (short form)	96	6:45
Multiplication	4/ 2/35	5A	100	3:40
	4/ 2/35	5P (short form)	100	13:00
Division	5/ 9/35	6A	100	3:27
	5/ 9/35	6P2	92	42:10

Summary of Case I. When Roland learned that he was to be given special help in arithmetic, he was more than pleased. During the course of the first interview he said that he was interested and willing to work in order that he might improve his work in arithmetic.

Two forty-five minute instructional periods were devoted to corrective work each week for approximately four months. The thirty teaching periods were divided among the processes approximately as follows: addition 7 periods; subtraction 5 periods; multiplication 6 periods; and division 12 periods.

The final results in Table II show a marked improvement was made with respect to speed and accuracy. In other words, Case I, at the end of the remedial instruction, worked more rapidly with no loss in accuracy.

A further evidence of gain is indicated from the child's own admission that he has more confidence in his work and results.

Throughout all these years he received only one poor passable grade in arithmetic and that one at the end of the fourth grade. Since then, and up to the time of this study, he has always received a failing grade in this subject.

Personality of Child. He is a pleasant, cooperative boy, somewhat shy, and lacking ambition. His general appearance is satisfactory, and his general conduct is satisfactory. He appears to be one of those children who are incapable of quiet action.

His general attitude is good. He is a quiet child, and extremely affected by adverse criticism from his classmates. He seeks to forget

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CASE II

Roy was in the sixth grade at the time this study was made. He was born in April, 1921. His I.Q. is 80 according to the Binet-Simon Intelligence Test.

He was referred for study by the principal of the Shurtleff School because he continued to fail in arithmetic.

School History. This child began school at the age of six, and throughout the earlier grades his school work in all phases, in general, was poor. Even now his work does not show any appreciable improvement.

He was not promoted at the end of the first year, instead he was transferred to a special class. His school record at the end of the second grade showed that his work was of an unsatisfactory grade, but it seems that he was promoted because of his age. Since then his work has improved very slightly. He is now in the sixth grade on trial.

Throughout all these years he received only one poor passable grade in arithmetic and that was at the end of the fourth grade. Since then, and up to the time of this study, he has always received a failing grade in this subject.

Personality of Child. He is a pleasant, cooperative boy, somewhat shy, and lacking ambition. His general appearance is satisfactory, and his general movements rather slow. He appears to be one of those children who are incapable of quick action.

His general attitude is good. He is a quiet child, and extremely effected by adverse criticism from his classmates. He seems to forget

things easily; his mental reactions appear to function very sluggishly.

He displays an inferiority complex, possibly because of his unsatisfactory school work. During the first interview he was told that he was to receive special help in arithmetic. He showed signs of contentment and eagerness to improve in the fundamental processes.

Home Conditions. He is the 7th child in the family in which there are 13 children. The oldest is 26 years old while the youngest is about 3 years old. There are six boys and seven girls in the family. An older sister is reported to be mentally retarded.

The father is employed in a shoe factory while the mother is engaged in home activities. A visit to the home showed that it was reasonably well kept, but the conditions of the house are such that there is much noise and the rooms are crowded because of the number of small children at home.

The family is not very cooperative with the school according to his teacher, but nevertheless interested in the child and sometimes helps him with arithmetic at home.

Results of Tests in Arithmetic. Table III gives Roy's initial results on the Wilson Inventory and Diagnostic Tests. The time taken to complete the tests is recorded in all but one case, that of the long form of the multiplication test. In cases where time is recorded it reads in minutes and seconds, for example, 3:10 is read 3 minutes and 10 seconds.

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TABLE III

Initial Results of Case II on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	11/5/34	3A	100	3:10
	11/5/34	3B	94	5:48
	11/5/34	3C	95+	25:00
	11/7/34	3P (short form)	76	10:15
Subtraction	11/19/34	4A	100	3:20
	11/19/34	4B	99	7:00
	12/ 4/34	4P (long form)	59	25:10
Multiplication	1/29/35	5A	97	3:40
	1/29/35	5P (long form)	66
Division	2/ 4/35	6A	94	18:10
	2/25/35	6P ₂	..	55:00

* Score not recorded because the entire test was not completed. The child attempted 41 examples out of a possible 50; 32 were correct. He was stopped because he was working them very slowly.

TABLE III

Initial Results of Case II on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Abstraction	11/8/34	3A	100	2:10
	11/8/34	3B	74	2:48
	11/8/34	3C	93*	23:00
	11/7/34	3P (short form)	78	10:15
Reproduction	11/13/34	4A	100	3:20
	11/13/34	4B	92	7:20
	12/4/34	4P (long form)	98	28:10
Intelligence	1/23/35	5A	97	2:40
	1/23/35	5P (long form)	98
Division	2/4/35	6A	94	18:10
	2/28/35	6P	..	23:01

* Score not recorded because the native fact was not completed. The child attempted 41 examples out of a possible 50; 32 were correct. He was dropped because he was working them very slowly.

Analysis of Difficulties. A careful diagnosis of Roy's habits of work in working the various examples in the tests revealed errors and inefficient habits of work. The diagnosis was based upon the results of the various test. The following tabulation shows the errors and the inefficient habits of work:

Addition

1. Errors in primary combinations
2. Errors in upper decade combinations
3. Counting
4. Split numbers
5. Added carried number last - added carried number irregularly
6. Grouped numbers

Subtraction

1. Errors in primary combinations
2. Last subtraction a zero brought down
3. Deducted two from minuend after borrowing
4. Did not allow for having borrowed
5. Failed to borrow, gave zero as an answer
6. Added instead of subtracting
7. Errors due to zero in minuend

Multiplication

1. Errors in combinations
2. Errors due to zero in multiplicand
3. Left out decimal point (U.S. Money)
4. Errors in adding the carried number

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1. Errors in primary combinations
2. Last subtraction a zero brought down
3. Borrowed two from tens and after borrowing
4. Did not allow for having borrowed
5. Failed to borrow, gave zero as an answer
6. Added instead of subtracting
7. Errors due to zero in minuend

Multiplication

1. Errors in combinations
2. Errors due to zero in multiplicand
3. Left out decimal point (U.S. Money)
4. Errors in adding the carried number

5. Errors due to zero in multiplier
6. Errors in position of partial products

Division

1. Errors in division combinations
2. Used remainder larger than divisor
3. Errors in subtraction
4. Errors in multiplication
5. Left out dollar sign and decimal point (U.S. Money)

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests are illustrated below. They were taken from the various tests and appear in the exact form.

Difficulties in Addition

1. Errors in primary combinations

The following primary combinations were missed:

$$\begin{array}{r} 7 \\ 9 \\ \hline 17 \end{array} \quad \begin{array}{r} 9 \\ 7 \\ \hline 15 \end{array} \quad \begin{array}{r} 9 \\ 4 \\ \hline 12 \end{array} \quad \begin{array}{r} 6 \\ 4 \\ \hline 11 \end{array} \quad \begin{array}{r} 4 \\ 6 \\ \hline 11 \end{array} \quad \begin{array}{r} 7 \\ 8 \\ \hline 14 \end{array}$$

2. Errors in upper decade combinations

The following upper decade combinations were missed:

$$38 + 6 = 34$$

$$38 + 7 = 35$$

$$23 + 9 = 30$$

$$19 + 5 = 34$$

$$33 + 9 = 48$$

$$29 + 7 = 26$$

$$26 + 9 = 33$$

$$19 + 8 = 24$$

$$32 + 7 = 49$$

$$34 + 6 = 4$$

$$22 + 7 =$$

$$33 + 5 = 40$$

$$17 + 8 = 23$$

3. Counting

Example:

$$\begin{array}{r} 6 \\ 5 \\ \hline 11 \end{array}$$

In working the example on the left he said, "6 and 5 are - 7, 8, 9, 10, 11," tapping each time with his finger when he said a new number. He counted the combinations he was not sure of, and counted the smaller number to the larger one.

4. Split numbers

Example:

$$\begin{array}{r} 7 \\ 5 \\ \hline 12 \end{array}$$

In this example, he borrowed 1 from 7 and added it to 5. Then he proceeded to obtain the answer for 6 and 6. Sometimes he broke numbers less than 10 into groups of two's or three's.

5. Added carried number last - added carried number irregularly

Example:

$$\begin{array}{r} 18 \\ 31 \\ 12 \\ 21 \\ 12 \\ \hline 94 \end{array}$$

This pupil usually waited to the last to add the carried number in column addition. He had no systematic plan of adding the carried number. In the example on the left he obtained the sum of the numbers in the first column, - then proceeded to obtain the sum of the numbers in the second column, and finally added the number to be carried. Because of this habit he made errors in other similar examples, which would probably have been avoided if he had habitually added the carried number first.

6. Grouped numbers

Example:

$$\begin{array}{r} \$3.79 \\ 8.94 \\ 3.48 \\ .86 \\ 3.95 \\ \hline \$20.92 \end{array}$$

In this example he grouped the 9 and 6 in the first column as 15, using it as a single addend, and then proceeded to add the rest of the figures in that column. He obtained an incorrect final sum.

3. Counting

Example:

6
3
11

In working the example on the left he said, "6 and 3 are - 9, 9, 10, 11," tapping each time with his finger when he said a new number. He counted the combinations he was not sure of, and counted the smaller number to the larger one.

4. Split numbers

Example:

7
3
10

In this example, he borrowed 1 from 7 and added it to 3. Then he proceeded to obtain the answer for 3 and 3. Sometimes he broke numbers less than 10 into groups of two's or three's.

5. Added carried number first - added carried number irregularly

Example:

18
31
12
21
18
94

This pupil usually waited to the last to add the carried number in column addition. He had no systematic plan of adding the carried number. In the example on the left he obtained the sum of the numbers in the first column, - then proceeded to obtain the sum of the numbers in the second column, and finally added the number to be carried. Because of this habit he made errors in other similar examples, which would probably have been avoided if he had habitually added the carried number first.

6. Grouped numbers

Example:

55.75
5.55
5.55
55
5.55
120.90

In this example he grouped the 5 and 5 in the first column as 10, adding it as a single addend, and then proceeded to add the rest of the figures in that column. He obtained an incorrect final sum.

Difficulties in Subtraction

1. Errors in primary combinations

The following primary combinations were missed

$$\begin{array}{r} 13 \\ 5 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 14 \\ 5 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 5 \\ 0 \\ \hline 0 \end{array}$$

Example:

$$\begin{array}{r} \$.98 \\ .50 \\ \hline \$.40 \end{array}$$

In this example the error was made in the first column.

2. Last subtraction a zero brought down

Example 1:

$$\begin{array}{r} 96 \\ 91 \\ \hline 05 \end{array}$$

In this example the zero in the last subtraction was brought down. Roy did this frequently.

Example 2:

$$\begin{array}{r} 489 \\ 453 \\ \hline 036 \end{array}$$

In this example the last subtraction a zero was brought down.

3. Deducted two from minuend after borrowing

Example:

$$\begin{array}{r} 98 \\ 79 \\ \hline 9 \end{array}$$

In this example, Roy apparently deducted 2 from the 9 instead of 1.

4. Did not allow for having borrowed

Example:

$$\begin{array}{r} 709 \\ 236 \\ \hline 573 \end{array}$$

This pupil worked the example as follows: "6 to make 9 is 3; 3 to make 10 is 7; 2 to make 7 is 5."

5. Failed to borrow, gave zero as an answer

Example:

$$\begin{array}{r} 600 \\ 140 \\ \hline 500 \end{array}$$

In this example he failed to borrow in the second column and gave zero as the answer.

6. Added instead of subtracting

Example 1:

$$\begin{array}{r} 71 \\ 62 \\ \hline 13 \end{array}$$

Roy added in the first column and subtracted in the second.

Example 2:

$$\begin{array}{r} \$42.95 \\ 31.40 \\ \hline \$13.50 \end{array}$$

In this example, he added in the third column and subtracted in all others.

7. Errors due to zero in the minuend

Example:

$$\begin{array}{r} 700 \\ 368 \\ \hline 442 \end{array}$$

In this example with a succession of zeros in the minuend, he forgot that he had borrowed. He worked the example as follows: "8 to make 10 is 2; 6 to make 10 is 4; 3 to make 7 is 4."

Difficulties in Multiplication

1. Errors in combinations

The following are some of the errors that the boy made:

$$\begin{array}{r} 9 \\ 6 \\ \hline 72 \end{array}$$

$$\begin{array}{r} 3 \\ 6 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 9 \\ 5 \\ \hline 54 \end{array}$$

2. Errors due to zero in multiplicand

Example:

$$\begin{array}{r} 408 \\ 4 \\ \hline 1902 \end{array}$$

In working this example he said, "4 times 8 equals 32; 4 times 0 equals 0; 4 times 4 equals 16 and 3 is 19."

3. Left out decimal point (U.S. Money)

Example:

$$\begin{array}{r} \$9.30 \\ 29 \\ \hline 83\ 70 \\ 186\ 0 \\ \hline \$269\ 70 \end{array}$$

In this example he forgot to mark off the decimal point. He was not consistent about it. Sometimes he marked off the decimal and other times he forgot to do it. He also sometimes left out the dollar sign.

4. Errors in adding the carried number

Example:

$$\begin{array}{r} 896 \\ 83 \\ \hline 2688 \\ 6168 \\ \hline 64368 \end{array}$$

This child multiplied by 8 as follows: "8 times 6 equals 48; 8 times 9 equals 72 and 4 is 76; 8 times 8 equals 64 and 7 is 61."

5. Errors due to zero in multiplier

Example:

$$\begin{array}{r} 915 \\ 504 \\ \hline 3660 \\ 45755 \\ \hline 461210 \end{array}$$

This child knew how to work single combinations involving zero correctly, but in working this example and similar ones he multiplied incorrectly. He worked this example as follows: "4 times 5 equals 20; 4 times 1 equals 4 and 2 is 6; 4 times 9 equals 36; 0 times 5 equals 5; 5 times 5 equals 25," etc.

6. Errors in position of partial products

Example:

$$\begin{array}{r} 842 \\ 2100 \\ \hline 84200 \\ 1684 \\ \hline 252600 \end{array}$$

In working this example, he placed the second product in the wrong position.

Division Difficulties

1. Errors in division combinations

The following were missed:

$$3) \overline{27}^{8, 3R} \quad 8) \overline{56}^8 \quad 7) \overline{28}^3 \quad 9) \overline{54}^5 \quad 1) \overline{2}^1 \quad 8) \overline{32}^3$$

4. Errors in adding the carried number

Example:

$$\begin{array}{r} 898 \\ 35 \\ \hline 933 \\ 8188 \\ \hline 90818 \end{array}$$
 This child multiplied by 8 as follows: "8 times 8 equals 64; 8 times 9 equals 72 and 6 is 78; 8 times 8 equals 64 and 7 is 71."

5. Errors due to zero in multiplier

Example:

$$\begin{array}{r} 918 \\ 308 \\ \hline 2754 \\ 27540 \\ \hline 284580 \end{array}$$
 This child knew how to work single combinations involving zero correctly, but in working this example and similar ones he multiplied incorrectly. He worked this example as follows: "4 times 8 equals 32; 4 times 1 equals 4 and 2 is 6; 4 times 9 equals 36; 0 times 8 equals 0; 0 times 3 equals 0; 0 times 2 equals 0, etc."

6. Errors in position of partial products

Example:

$$\begin{array}{r} 845 \\ 2100 \\ \hline 17700 \\ 1684 \\ \hline 176800 \end{array}$$
 In working this example, he placed the second product in the wrong position.

Division Difficulties

1. Errors in division combinations

The following were missed:

$$\begin{array}{l} 8.25 \\ 3 \overline{) 25} \\ 24 \\ \hline 10 \\ 9 \\ \hline 1 \\ 0 \\ \hline 0 \end{array}$$

2. Used remainder larger than divisor

Example:

$$\begin{array}{r}
 36 \quad 126/63 \\
 63 \overline{) 2394} \\
 \underline{189} \\
 504 \\
 \underline{378} \\
 126
 \end{array}$$

In this example, Roy used the last remainder larger than the divisor.

3. Errors in subtraction

Example:

$$\begin{array}{r}
 91 \\
 47 \overline{) 4300} \\
 \underline{423} \\
 70 \\
 \underline{47} \\
 33
 \end{array}$$

In subtracting at the point indicated, he forgot that he had borrowed.

4. Errors in multiplication

Example:

$$\begin{array}{r}
 225 \\
 31 \overline{) 6874} \\
 \underline{61} \\
 77 \\
 \underline{62} \\
 154 \\
 \underline{155} \\
 1
 \end{array}$$

The pupil worked as shown.

5. Left out dollar sign and decimal point (U.S. Money)

Example:

$$\begin{array}{r}
 70 \\
 45 \overline{) \$31.50} \\
 \underline{315} \\
 0
 \end{array}$$

Roy worked this example as shown. He left out the dollar sign and decimal point.

General Plan of Remedial Instruction. The purpose of remedial instruction was to remove the handicaps which faced the boy in his attempts to deal with the four fundamental processes of arithmetic. The attack was primarily concerned with better presentation of the processes and facts, and with drill on the processes and facts after they have been presented.

Before initiating remedial instruction a friendly relationship was established so that it might be possible to awaken an interest in the work and to enlist his cooperation. After this friendly relationship was established the test results were shown to him, and he was asked to demonstrate his methods of thinking through the incorrect and the questionable examples by working them aloud. The disadvantages of crude methods were explained and substituted by methods considered more efficient. To engender more efficient habits of work the necessary practice material was furnished from the drill services. Addition was the first process taken up, it was followed by subtraction, multiplication and division respectively.

During the instructional periods the work was carefully watched and constantly checked for correct answers and for desirable habits of work. Frequently he was asked to work the examples aloud so that his methods of thinking through such examples might be checked. All undesirable habits of work were immediately called to his attention. They were corrected, noted, and carefully watched for reappearance. During these periods satisfactory work was commended. For overcoming faulty habits of work he was praised and encouraged to keep up the good work.

In order to build confidence in his results he was shown how to check examples, and was encouraged to check them from time to time. During the

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course of the remedial program many informal examinations were given. The purpose of such examinations was to check for mastery and to discover whether or not faulty habits fell into disuse.

Detailed Account of Remedial Instruction. The paragraphs below explain the methods employed to improve the child's habits of work. A description is given of the remedial instruction employed in the cases of the various difficulties discovered. Reference to the list of the detailed description of habits given in the early part of this case study will enable the reader to identify the habits being considered at any time.

Addition

Flash cards were constructed and used to overcome the errors in the primary combinations in each of the four fundamental processes. He was required to study them. In testing for mastery he gave the answers aloud and was expected to respond with a reasonable degree of speed and accuracy. Whenever he hesitated too long on a combination, it was an indication that he had not mastered it. Such combinations were set aside for further study and placed in his line of vision so that the combination could be called to his attention whenever it reappeared in an example during that instructional period.

After having mastered the primary facts within the group, he studied the related facts of that group. These facts were studied from individual flash cards in much the same way as the primary facts.

He did not count all his combinations, but some of them which he referred to as difficult. Such a list was not obtained. It was decided as unnecessary because he was expected to study the facts as grouped in the

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Additional

Flash cards were constructed and used to overcome the errors in the primary combinations in each of the four fundamental processes. He was required to study them. In testing for mastery he gave the answers aloud and was expected to respond with a reasonable degree of speed and accuracy. Whenever he hesitated too long on a combination, it was an indication that he had not mastered it. Such combinations were set aside for further study and placed in his line of vision so that the combination could be called to his attention whenever it reappeared in an example during that instructional period.

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Wilson 100% Drill Services. After several instructional periods he gradually became more communicative. Whenever he hesitated, he was asked if he counted, and he always replied. When he admitted that he counted, the fact was set aside for further practice. He studied the facts until he mastered them.

After having mastered the primary facts and related facts in a group, the habit of splitting numbers was not needed and finally fell into disuse.

The possible outcomes of such a procedure as adding the carried number last or irregularly were illustrated by concrete examples. In attempting to overcome this habit, the better procedure was illustrated - adding the carried number first; He was requested to work next example in the book and after he added the first column he was stopped and told to add in the carried number at the bottom of the second column. By repeating such a plan orally many times, the habit of adding the carried number first was firmly impressed.

He did not group numbers after he had mastered the primary and related facts, and instead added the numbers in a column in a successive order, because he knew them.

Subtraction

In examples in which the last subtraction was a zero and he brought down the zero, he was told that the zero in such cases was not necessary. Then, he was asked to read the answer followed by a request to write the same number. The result was that he wrote the number without the zero. This was sufficient to convince him that the zero in such cases was not necessary.

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Subtraction

In examples in which the last subtraction was a zero and he brought down the zero, he was told that the zero in such cases was not necessary. Then, he was asked to read the answer followed by a request to write the same number. The result was that he wrote the number without the zero. This was sufficient to convince him that the zero in such cases was not necessary.

Subtraction habits 3, 4, 5, and 7, were corrected by having the child cross out the number whenever he had borrowed from it and to write directly above it the number that was left. At first he disliked doing this and in fact insisted upon not doing it. He said that he could get the correct answer without such a plan. Under the circumstances he was allowed to work his way. Sometimes he obtained the correct answer, but in most cases errors appeared because of these difficulties. In such cases he was requested to try the same example by the new plan and found that he obtained the correct answer. Gradually he realized the advantages of the new plan and soon followed it regularly. The result was that the correct habits were established.

Sometimes he added instead of subtracting. This habit might have been attributed to carelessness. However, when an error arose because of this habit, his attention was called to it, and he seemed surprised and could not explain it. He was always requested to make the correction whenever such an error appeared.

Multiplication

To correct the difficulty due to zero in the multiplicand, I showed the child the mistake and illustrated the correct procedure. This instruction was followed by drill in similar practice exercises.

To correct the habit of leaving out the decimal point in U.S. money further drill was provided.

Whenever an answer was incorrect due to an error in adding the carried number, his attention was called to the mistake. He was told that the multiplication was correct, but that the answer was wrong due to

an error in adding the carried number. He was encouraged to be more careful.

The mistakes due to such factors as are listed under multiplication difficulties 5, and 6 were corrected by a presentation of the correct methods and further drill on similar exercises.

Division

To overcome division habits 2 and 5, the mistakes were pointed out and the correct procedures illustrated. Following this, practice exercises were assigned to insure mastery.

Whenever division habits 3 and 4 appeared, he was requested to do that part of the example aloud, and make the correction.

Final Test Results. At the conclusion of the remedial instruction the Wilson Inventory and Diagnostic Tests were administered in each of the fundamental processes. The results of the final tests are listed in Table IV on the following page. Tests 3C and 6P₂ were given in two and three installments respectively. In such cases the date of the first installment appears, the total score, and the total time taken to complete the entire test.

TABLE IV

Final Results of Case II on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	3/15/35	3C	98	16:10
	1/ 4/55	3P (short form)	96	9:00
Subtraction	3/11/35	4A	100	3:00
	3/11/35	4B	100	6:41
	3/18/35	4P (short form)	92	4:47
Multiplication	3/20/35	5A	100	2:40
	3/27/35	5P (short form)	88	8:45
Division	4/ 8/35	6A	100	5:45
	4/ 8/35	6Pg	90	38:40

TABLE IV

Final Results of Case II on the Wilson Inventory
and Magnetic Tests

Process	Date	Test	Score	Time
Addition	3/15/33	33	33	13:13
	1/4/33	33 (short form)	33	3:00
Subtraction	3/11/33	34	100	3:00
	3/11/33	33	100	3:41
	3/18/33	33 (short form)	33	4:57
Multiplication	3/20/33	34	100	3:40
	3/27/33	33 (short form)	33	3:40
Division	4/8/33	33	100	3:43
	4/8/33	33	33	33:40

Summary of Case II. When this study began Roy knew that he was weak in arithmetic. During the interview he said that he was interested and would be willing to work so that he could make a good showing in class.

Three forty-five minute instructional periods were devoted to corrective work each week for approximately five months. There were about forty-eight teaching periods divided among the processes approximately as follows: addition 12 periods; subtraction 10 periods; multiplication 11 periods; and division 15 periods.

The final results in Table IV show that the remedial instruction produced distinctly favorable results. Further evidence of gain in arithmetic was reported by his teacher who claimed that Roy's work has improved and that he is more interested than before. The principal also reported that the boy now displays a better attitude toward his work and that he shows general improvement. Roy reports that he feels that he has learned a lot and that he is more confident in his work.

After attending the public school for several months, his teacher reported that he was now doing well in his subjects, and that his greatest difficulty was arithmetic. She reported that in arithmetic, he could not complete the assigned arithmetic work, and he was always last among the other members of his class.

Personality of Child. He is quiet, pleasant, and very well liked by his classmates. At times he appears to be nervous. During the first periods he seemed nervous, and gave the impression that he was working under pressure. During the instructional periods he paid strict attention, and followed directions without exception.

Summary of Case 11. When this study began, Roy knew that he was weak in arithmetic. During the interview he said that he was interested and would be willing to work so that he could make a good showing in class.

Three forty-five minute instructional periods were devoted to carrying five work each week for approximately five months. There were about forty-eight teaching periods divided among the processes approximately as follows: addition 15 periods; subtraction 10 periods; multiplication 11 periods; and division 15 periods.

The final results in Table 19 show that the remedial instruction produced distinctly favorable results. Further evidence of gain in arithmetic was reported by his teacher who claimed that Roy's work has improved and that he is more interested than before. The principal also reported that the boy now displays a better attitude toward his work and that he shows general improvement. Roy reports that he feels that he has learned a lot and that he is more confident in his work.

CASE III

At the time this study was made Arman was in the fifth grade. He was born in April, 1924. His intelligence quotient was found to be 99 according to the results of the Otis Group Intelligence Test.

He was referred for study by the principal of the Shurtleff School because his classroom teacher reported that he was doing very poor work in arithmetic.

School History. Arman entered a parochial school at the age of five. During the first school year he was ill most of the time and absent from school. At the end of the first year he was not promoted and had to repeat that grade. He attended this school for five years. After completing the fourth grade, his parents transferred him to a public school because of some difficulty that arose between the parents and the administrators of the school.

After attending the public school for several months, his teacher reported that he was not doing well in his subjects, and that his greatest difficulty was arithmetic. She reported that in arithmetic, he could never complete the assigned classroom work, and he was always far behind the other members of his class.

Personality of Child. He is quiet, pleasant, and very neat in his appearance. At times he appears to be nervous. During the test periods he moved around, and gave the impression that he was working under pressure. During the instructional periods he paid strict attention, and followed directions rather carefully.

At the time this study was made Arman was in the fifth grade. He was born in April, 1934. His intelligence quotient was found to be 99 according to the results of the Otis Group Intelligence Test.

He was referred for study by the principal of the Sharpley School because his classroom teacher reported that he was doing very poor work in arithmetic.

School History. Arman entered a parochial school at the age of five. During the first school year he was ill most of the time and absent from school. At the end of the first year he was not promoted and had to repeat that grade. He attended this school for five years. After completing the fourth grade, his parents transferred him to a public school because of some difficulty that arose between the parents and the administrators of the school.

After attending the public school for several months, his teacher reported that he was not doing well in his subjects, and that his greatest difficulty was arithmetic. She reported that in arithmetic, he could never complete the assigned classroom work, and he was always far behind the other members of his class.

Personality of Child. He is quiet, pleasant, and very neat in his appearance. At times he appears to be nervous. During the test periods he moved around, and gave the impression that he was working under pressure. During the instructional periods he paid strict attention, and followed directions rather carefully.

Arman is cooperative and faithful. He dislikes going over the same assignments. In class he is very quiet and seldom volunteers. He is neither aggressive nor talkative.

Home Conditions. There are two children in the family. Arman is the older. The younger boy is nine and in the second grade in the same school.

Both parents work in a shoe factory doing specialized operations. The mother is very much interested in Arman. She helps him a great deal with arithmetic at home. He also practices daily on his violin.

A visit to the home was not made, but from the general appearance and attitude of this boy it seems that his home conditions are satisfactory. The home is located in the proximity of the school.

Results of Tests in Arithmetic. Table V gives Arman's initial results on the Wilson Inventory and Diagnostic Tests. The time to complete such tests is recorded in every case. Test 3C was given in two installments. In that case the date of the first installment appears, the total score, and the total time taken to complete the entire test. The time reads in minutes and seconds, for example, 9:30 for test 3A reads 9 minutes and 30 seconds.

Armen is cooperative and friendly. He dislikes going over the team assignments. In class he is very quiet and seldom volunteers. He is neither aggressive nor talkative.

Home Conditions. There are two children in the family. Armen is the older. The younger boy is nine and in the second grade in the same school. Both parents work in a shoe factory doing specialized operations. The mother is very much interested in Armen. She helps him a great deal with arithmetic at home. He also practices daily on his violin. A visit to the home was not made, but from the general appearance and attitude of this boy it seems that his home conditions are satisfactory. The home is located in the proximity of the school.

Results of Tests in Arithmetic. Table V gives Armen's initial results on the Wilson Inventory and Diagnostic Tests. The time to complete each test is reported in every case. Test 33 was given in two installments. In that case the date of the first installment appears, the total score, and the total time taken to complete the entire test. The time needed in minutes and seconds, for example, 9:30 for test 3A reads 9 minutes and 30 seconds.

TABLE V

Initial Results of Case III on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	1/10/35	3A	99	9:30
	1/10/35	3B	100	13:32
	1/10/35	3C	98	57:02
	1/14/35	3P (short form)	84	31:35
Subtraction	1/16/35	4A	100	8:00
	1/16/35	4B	100	17:31
	1/16/35	4P (long form)	85	40:15
Multiplication	1/21/35	5A	98	29:30
	1/25/35	5P (short form)	24	28:00
Division	4/30/35	6A	96	10:53
	4/29/35	6P ₂ *	55	43:28

* Test 6P₂* is an abbreviated form of the 6P₂ test. It contains twenty long division examples, two from each group in the 6P₂ test. The writer received the author's permission.

TABLE V

Initial Results of Case III on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
addition	1/10/38	5A	99	2:30
	1/10/38	5B	100	13:35
	1/10/38	5C	98	37:30
	1/14/38	5P (short form)	94	31:30
subtraction	1/10/38	6A	100	3:00
	1/10/38	6B	100	17:51
	1/10/38	6P (long form)	95	40:15
multiplication	1/21/38	8A	98	23:30
	1/23/38	8P (short form)	96	28:00
division	2/20/38	9A	96	10:30
	4/22/38	9P*	99	43:30

* Test 9P* is an abbreviated form of the 9P test. It contains twenty long division exercises, two from each group in the 9P test. The writer received the author's permission.

Analysis of Difficulties. A careful diagnosis of Arman's habits of work revealed errors and inefficient habits of work. The diagnosis was based upon the results of the various tests. The following tabulation shows the errors and the inefficient habits of work:

Addition

1. Error in a primary combination
2. Errors in upper decade combinations
3. Counting
4. Added carried number irregularly
5. Left out dollar sign (U.S. Money)
6. Error in bridging

Subtraction

1. Added instead of subtracting
2. Omitted column
3. Did not allow for having borrowed
4. Errors due to a succession of zeros in minuend

Multiplication

1. Errors in primary combinations
2. Errors in adding the carried number
3. Complete confusion with multipliers of two or more digits

Division

1. Errors in division combinations
2. Used remainder larger than divisor
3. Errors in subtraction
4. Omitted digit in dividend
5. Remainder not expressed in answer

Analysis of Difficulties. A careful diagnosis of Arden's habits of work revealed errors and inefficient habits of work. The diagnosis was based upon the results of the various tests. The following tabulation shows the errors and the inefficient habits of work:

Addition

1. Error in a primary combination
2. Errors in upper decade combinations
3. Carrying
4. Added carried number irregularly
5. Left out dollar sign (U.S. Money)
6. Error in bridging

Subtraction

1. Added instead of subtracting
2. Omitted column
3. Did not allow for having borrowed
4. Errors due to a succession of zeros in minuend

Multiplication

1. Errors in primary combinations
2. Errors in adding the carried number
3. Complete combination with multipliers of two or more digits

Division

1. Errors in division combinations
2. Used remainder larger than divisor
3. Errors in subtraction
4. Omitted digit in dividend
5. Remainder not expressed in answer

6. Confused by dollar sign and decimal point
7. Used remainder without new dividend figure

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests are illustrated below. They are shown in the same form as they appear in the original test.

Difficulties in Addition

1. Error in a primary combination

The following primary combination was missed

$$\begin{array}{r} 5 \\ 3 \\ \hline 4 \end{array}$$

2. Errors in upper decade combinations

The following upper decade combinations were missed:

$$24 + 9 = 32$$

$$38 + 5 = 33$$

$$16 + 9 = 27$$

$$24 + 7 = 3$$

$$33 + 8 = 42$$

$$33 + 7 = 39$$

3. Counting

Example:

$$\begin{array}{r} 7 \\ 4 \\ \hline 11 \end{array}$$

In working this example he counted the 4 to 7 by one's. When questioned about counting, he said that he counted the harder ones.

4. Added carried number irregularly

Example:

$$\begin{array}{r} 367 \\ 298 \\ 198 \\ \hline 863 \end{array}$$

In the second column he added the two that was carried from the first column to the sum of $9 + 9$ in that column, and continued from there on.

When questioned about the procedure he used in adding carried numbers, he said that he sometimes added the carried number first, sometimes within the column, and sometimes at the end.

6. Combined by dollar sign and decimal point
7. Used remainder without new dividend figure

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests are illustrated below. They are shown in the same form as they appear in the original test.

Difficulties in Addition

1. Error in a primary combination
The following primary combination was missed

$$\begin{array}{r} 3 \\ 2 \\ 4 \end{array}$$

2. Errors in upper decade combinations
The following upper decade combinations were missed:

$24 + 8 = 32$	$38 + 5 = 43$
$18 + 9 = 27$	$31 + 7 = 38$
$32 + 8 = 40$	$25 + 7 = 32$

3. Counting
Example:

7	In working this example he counted the 4 to 7 by one's.
4	When questioned about counting, he said that he counted
11	the harder ones.

4. Added carried number irregularly
Example:

287	In the second column he added the two that was carried
198	from the first column to the sum of 8 + 9 in that column,
138	and continued from there on.
585	When questioned about the procedure he used in adding
	carried numbers, he said that he sometimes added the
	carried number first, sometimes within the column, and
	sometimes at the end.

5. Left out dollar sign (U.S. Money)

Example:

$$\begin{array}{r}
 \$.55 \\
 5.39 \\
 4.87 \\
 2.09 \\
 9.75 \\
 \hline
 23.55
 \end{array}$$

In this example, he left out the dollar sign.
The addition in the second column is also incorrect.

6. Error in bridging

Example:

$$\begin{array}{r}
 \$1.20 \\
 .54 \\
 6.65 \\
 9.50 \\
 2.17 \\
 \hline
 \$30.06
 \end{array}$$

In adding the third column, he skipped 10.

Difficulties in Subtraction

1. Added instead of subtracting

Example:

$$\begin{array}{r}
 638 \\
 102 \\
 \hline
 736
 \end{array}$$

In working this example, he added the last column instead of subtracting.

2. Omitted column

Example:

$$\begin{array}{r}
 156 \\
 42 \\
 \hline
 4
 \end{array}$$

In this example the tens' and hundreds' columns were omitted.

3. Did not allow for having borrowed

Example:

$$\begin{array}{r}
 501 \\
 163 \\
 \hline
 348
 \end{array}$$

In working this example, he did not allow for having borrowed in the second column.

2. Left out dollar sign (U.S. money)

Example:

\$	5.00
5.00	
5.00	
5.00	
5.00	
5.00	
5.00	
5.00	
5.00	
5.00	

In this example, he left out the dollar sign. The addition in the second column is also incorrect.

3. Error in bridging

Example:

\$	1.00
1.00	
1.00	
1.00	
1.00	
1.00	
1.00	
1.00	
1.00	
1.00	

In adding the third column, he skipped 10.

Difficulties in subtraction

1. Added instead of subtracting

Example:

500	
100	
100	
100	
100	
100	
100	
100	
100	
100	

In working this example, he added the last column instead of subtracting.

2. Omitted column

Example:

100	
40	
40	
40	
40	
40	
40	
40	
40	
40	

In this example the tens' and hundreds' columns were omitted.

3. Did not allow for having borrowed

Example:

100	
100	
100	
100	
100	
100	
100	
100	
100	
100	

In working this example, he did not allow for having borrowed in the second column.

4. Errors due to a succession of zeros in minuend

Example 1:

$$\begin{array}{r} 600 \\ 149 \\ \hline 461 \end{array}$$

In working examples with a succession of zeros, he forgot that he had borrowed.

Example 2:

$$\begin{array}{r} \$2.00 \\ .75 \\ \hline \$1.35 \end{array}$$

This example also illustrates the same point.

Difficulties in Multiplication

1. Errors in primary combinations

The following primary combinations were missed:

$$\begin{array}{r} 8 \\ 6 \\ \hline 64 \end{array} \quad \begin{array}{r} 6 \\ 8 \\ \hline 64 \end{array}$$

2. Errors in adding the carried number

Example:

$$\begin{array}{r} \$3.65 \\ 6 \\ \hline \$20.90 \end{array}$$

In working this example, he made a mistake in adding the carried number.

3. Complete confusion with multipliers of two or more digits

Example:

$$\begin{array}{r} 92 \\ 56 \\ \hline 46,542 \end{array}$$

He was completely confused as to the method to employ when the multiplier had two digits or more.

Difficulties in Division

1. Errors in division combinations

The following division combinations were missed:

$$2 \overline{) 20 \over 10}$$

$$5 \overline{) 100 \over 20}$$

$$9 \overline{) 4 \over 63}$$

$$9 \overline{) 6 \over 45}$$

4. Errors due to a succession of errors in addition

Example 1:

$$\begin{array}{r} 800 \\ 148 \\ \hline 948 \end{array}$$
 In working examples with a succession of errors, he forgot that he had borrowed.

Example 2:

$$\begin{array}{r} 82.00 \\ 72. \\ \hline 154.00 \end{array}$$
 This example also illustrates the same point.

Difficulties in Multiplication

1. Errors in primary combinations

The following primary combinations were missed:

$$\begin{array}{r} 8 \\ 8 \\ \hline 64 \end{array}$$

2. Errors in adding the carried number

Example:

$$\begin{array}{r} 82.88 \\ 8 \\ \hline 82.96 \end{array}$$
 In working this example, he made a mistake in adding the carried number.

3. Complete confusion with multipliers of two or more digits

Example:

$$\begin{array}{r} 52 \\ 52 \\ \hline 2704 \end{array}$$
 He was completely confused as to the method to employ when the multiplier had two digits or more.

Difficulties in Division

1. Errors in division combinations

The following division combinations were missed:

$$\begin{array}{r} 20 \\ 2 \overline{) 40} \\ \hline 20 \end{array}$$

$$\begin{array}{r} 100 \\ 2 \overline{) 200} \\ \hline 100 \end{array}$$

$$\begin{array}{r} 4 \\ 2 \overline{) 8} \\ \hline 4 \end{array}$$

$$\begin{array}{r} 8 \\ 2 \overline{) 16} \\ \hline 8 \end{array}$$

2. Used remainder larger than divisor

Example:

$$\begin{array}{r}
 219 \quad 21/32 \\
 32 \overline{) 769} \\
 \underline{64} \\
 *62 \\
 \underline{32} \\
 309 \\
 \underline{288} \\
 21
 \end{array}$$

In working this example, he subtracted incorrectly and used this incorrect remainder which was larger than the divisor. The position is indicated by the asterisk.

3. Errors in subtraction

Example:

$$\begin{array}{r}
 93 \quad 2/98 \\
 98 \overline{) 9016} \\
 \underline{882} \\
 296 \\
 \underline{294} \\
 2
 \end{array}$$

In the first subtraction, he forgot that he had borrowed.

4. Omitted digit in dividend

Example:

$$\begin{array}{r}
 2 \\
 42 \overline{) 1008} \\
 \underline{84} \\
 26
 \end{array}$$

This pupil worked as shown. He failed to use the last 8 in the dividend. He also subtracted incorrectly, and did not express the remainder in the final answer.

5. Remainder not expressed in answer

Example:

$$\begin{array}{r}
 32 \\
 61 \overline{) 1974} \\
 \underline{183} \\
 144 \\
 \underline{122} \\
 22
 \end{array}$$

This pupil worked as shown, he failed to express the final remainder in the answer although he expressed it in other examples.

6. Confused by dollar sign and decimal point

Example:

$$\begin{array}{r}
 7 \\
 45 \overline{) \$31.50} \\
 \underline{315} \\
 .00
 \end{array}$$

This pupil worked as shown, he said that he never had such examples before.

2. Used remainder larger than divisor

Example:

$$\begin{array}{r} 21 \overline{) 32} \\ 21 \\ \hline 11 \\ 10 \\ \hline 2 \\ 2 \\ \hline 0 \end{array}$$

In working this example, he subtracted incorrectly and used this incorrect remainder which was larger than the divisor. The position is indicated by the asterisk.

3. Errors in subtraction

Example:

$$\begin{array}{r} 21 \overline{) 32} \\ 21 \\ \hline 11 \\ 10 \\ \hline 2 \\ 2 \\ \hline 0 \end{array}$$

In the first subtraction, he forgot that he had borrowed.

4. Omitted digit in dividend

Example:

$$\begin{array}{r} 21 \overline{) 32} \\ 21 \\ \hline 11 \\ 10 \\ \hline 2 \\ 2 \\ \hline 0 \end{array}$$

This pupil worked as shown. He failed to use the last 2 in the dividend. He also subtracted incorrectly, and did not express the remainder in the final answer.

5. Remainder not expressed in answer

Example:

$$\begin{array}{r} 21 \overline{) 32} \\ 21 \\ \hline 11 \\ 10 \\ \hline 2 \\ 2 \\ \hline 0 \end{array}$$

This pupil worked as shown. He failed to express the final remainder in the answer although he expressed it in other examples.

6. Confused by dollar sign and decimal point

Example:

$$\begin{array}{r} 21 \overline{) 32} \\ 21 \\ \hline 11 \\ 10 \\ \hline 2 \\ 2 \\ \hline 0 \end{array}$$

This pupil worked as shown. He said that he never had such examples before.

7. Used remainder without new dividend figure

Example:

$$\begin{array}{r}
 47 \overline{) 1598} \\
 \underline{141} \\
 188 \\
 \underline{141} \\
 47 \\
 \underline{47} \\
 0
 \end{array}$$

In working this example, he left too large a remainder after the second multiplication and then used it to get his third digit in the quotient.

General Plan of Remedial Instruction. The purpose of remedial instruction in this Case as in the other case studies was to remove the specific difficulties and to engender good habits of work in those phases of arithmetic which were discovered to be faulty. The attack was primarily concerned with better presentation of the facts and step difficulties within the processes, and followed by drill on such facts and step difficulties.

The diagnosis of the methods of work revealed that Arman was slow in his work, and that he had specific difficulties in each of the processes of arithmetic. To speed up his work, he competed with another boy, Dann, who was also deficient in the four fundamentals and who was receiving special help from the writer.

Throughout the corrective program the difficulties were treated when they arose during the instructional periods. Such a procedure was employed during the entire program of remedial instruction. When a difficulty appeared, it was immediately corrected, explained, and illustrated by a better method. Errors were never allowed to pass unnoted.

The work was carefully watched and constantly checked for correct answers and for desirable habits of work. Sometimes the boy was requested to repeat an example aloud in order that his methods of thinking through

Example:

$$\begin{array}{r} 1231 \\ 1238 \overline{) 1541} \\ \underline{1238} \\ 303 \\ \underline{303} \\ 0000 \end{array}$$

In working this example, he left too large a remainder after the second multiplication and then used it as his third digit in the quotient.

General Plan of Remedial Instruction. The purpose of remedial instruction

in this case as in the other case studies was to remove the specific difficulties and to engender good habits of work in these phases of arithmetic.

The work was discovered to be faulty. The attack was primarily concerned with better presentation of the facts and step difficulties within the processes, and followed by drill on each fact and step difficulties.

The diagnosis of the methods of work revealed that Arman was slow in his work, and that he had specific difficulties in each of the processes of arithmetic. To speed up his work, he competed with another boy, Dan, who was also deficient in the four fundamentals and who was receiving special help from the writer.

Throughout the corrective program the difficulties were treated when they arose during the instructional periods. Such a procedure was employed during the entire program of remedial instruction. When a difficulty appeared, it was immediately corrected, explained, and illustrated by a better method. Errors were never allowed to pass unnoted.

The work was carefully watched and constantly checked for correct answers and for desirable habits of work. Sometimes the boy was requested to repeat an example aloud in order that his methods of thinking through

the example might be checked.

In working the exercises in the 100% Drill Services, he worked in cooperation with Dann. After completing a group of examples, they compared answers and checked over again those that did not compare. In some cases the error was corrected by the writer, in other instances one of the boys was asked to show the other where the mistake was made and possibly why. The boy who obtained the correct answer was asked to explain why the mistake was made. They enjoyed such a plan very much.

Detailed Account of Remedial Instruction. The following paragraphs contain descriptions of the methods employed in correcting the specific difficulties that the child encountered in working with the processes. Reference to the list of the detailed description of habits given in the earlier part of this Case will enable the reader to identify the habits being considered at any time.

Addition

To overcome the errors in primary combinations, drill was employed. Flash cards were used. The child studied them in groups and in the same grouping as listed in the 100% Drill Services. He was tested on such facts during the instructional periods, if he failed or responded slowly to any of the facts he was expected to study them until he mastered them. Following this check, he was permitted to go on with the work.

Flash cards were also used to overcome errors in the upper decade facts.

During the corrective program, he worked on examples that contained

the examples might be checked.

In working the exercises in the 1000 Drill Exercises, he worked in cooperation with them. After completing a group of examples, they compared answers and checked over again those that did not compare. In some cases the error was corrected by the writer, in other instances one of the boys was asked to show the other where the mistake was made and possibly why. The boy who obtained the correct answer was asked to explain why the mistake was made. They enjoyed such a plan very much.

Detailed Account of Remedial Instruction. The following paragraphs contain descriptions of the methods employed in correcting the specific difficulties that the child encountered in working with the processes. References to the list of the detailed description of habits given in the earlier part of this case will enable the reader to identify the habits being considered at any time.

Attention

To overcome the errors in primary combinations, drill was employed. Flash cards were used. The child studied them in groups and in the same grouping as listed in the 1000 Drill Exercises. He was tested on each fact during the instructional periods, if he failed or responded slowly to any of the facts he was expected to study them until he mastered them. Following this check, he was permitted to go on with the work.

Flash cards were also used to overcome errors in the upper decade.

Tests.

During the corrective program, he worked on examples that contained

facts which he had studied and mastered. Such a plan resulted in the breaking up the habit of counting.

He added the carried number irregularly and as a result the final answer was incorrect many times. To overcome this habit the possible outcomes of such a habit were explained and illustrated by specific examples, followed by instruction to add the carried number first. From time to time he was checked orally to insure that he employed the new habit.

To overcome the habit of leaving out the dollar sign, the use of the dollar sign was explained and his work was carefully watched for the reappearance of this habit. Whenever it reappeared his attention was immediately called to it, and he was requested to make the correction in every such case.

To overcome the error of bridging, the nature of the mistake was first explained, and in every later case whenever it reappeared he was requested to add the example over again and aloud.

Subtraction

Even though he received perfect scores on tests of subtraction facts, he studied them from individual flash cards to increase his response to such facts.

To overcome subtraction habits 1 and 2, his work was carefully watched for the reappearance of those habits. When they reappeared, he was requested to make the necessary corrections.

To overcome the habit for not allowing for having borrowed, the error was explained very carefully. The subtraction was illustrated column by column. Following this, practice exercises were assigned and future work

facts which he had studied and mastered. Such a plan resulted in the

breaking up the habit of reasoning.

He added the carried number intelligently and as a result the final an-

swer was incorrect many times. To overcome this habit the possible over-

comes of such a habit were explained and illustrated by specific examples,

followed by instruction to add the carried number first. From time to

time he was checked orally to insure that he employed the new habit.

To overcome the habit of leaving out the dollar sign, the use of the

dollar sign was explained and his work was carefully watched for the re-

appearance of this habit. Whenever it reappeared his attention was imme-

diately called to it, and he was requested to make the correction in every

such case.

To overcome the error of doubling, the nature of the mistake was

first explained, and in every later case whenever it reappeared he was re-

quested to add the example over again and aloud.

Distraction

Even though he received perfect scores on tests of attention facts,

he studied them from individual flash cards to increase his response to

such facts.

To overcome distraction habits 1 and 2, his work was carefully watched

for the reappearance of those habits. When they reappeared, he was re-

quested to make the necessary corrections.

To overcome the habit of not allowing for having borrowed, the error

was explained very carefully. The subtraction was illustrated column by

column. Following this, practice exercises were assigned and future work

watched very carefully.

To overcome the difficulties due to a succession of zero in minuend, the subtraction was illustrated column by column. Practice exercises followed and work carefully watched.

Multiplication

To overcome the errors in primary combinations, flash cards were used. A similar procedure was employed as in addition.

To overcome errors due to adding incorrectly the carried number, the mistake in such instances was called to his attention and in every case he was requested to make the correction.

This child was completely confused when the multiplier had two or more digits. To clear up his confusion, the correct method was carefully explained and illustrated. Following that, practice exercises were assigned. Occasionally he made mistakes in the method, in such cases the mistake was corrected and explained by the writer.

Division

Division was not treated as intensively as the other fundamental processes because there remained very little for corrective.

The diagnosis of Arman's work revealed that he was not entirely ignorant of the procedure in division, but he made errors because of certain difficulties and faulty habits of work. In view of these factors, long division was reintroduced step by step to insure understanding.

Simple examples were first introduced, followed by drill to engender automatic procedure. The work was taken up as if he had not had long di-

watched very carefully.

To overcome the difficulties due to a succession of zero in minuend, the subtraction was illustrated column by column. Practice exercises followed and work carefully watched.

Multiplication

To overcome the errors in primary combinations, flash cards were used. A similar procedure was employed as in addition. To overcome errors due to adding incorrectly the carried number, the mistake in such instances was called to his attention and in every case he was requested to make the correction.

This child was completely confused when the multiplier had two or more digits. To clear up his confusion, the correct method was carefully explained and illustrated. Following that, practice exercises were assigned. Occasionally he made mistakes in the method, in such cases the mistake was corrected and explained by the writer.

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Division was not treated as intensively as the other fundamental processes because there remained very little for corrective. The diagnosis of Arman's work revealed that he was not entirely ignorant of the procedure in division, but he made errors because of certain difficulties and faulty habits of work. In view of these factors, long division was reintroduced step by step to insure understanding. Simple examples were first introduced, followed by drill to engender automatic procedure. The work was taken up as if he had not long di-

vision before. The various step difficulties were introduced step by step. They were illustrated and explained to prevent reoccurrence of old errors.

Final Test Results. At the conclusion of the remedial instruction, the Wilson Inventory and Diagnostic Tests were administered. The results of the final tests are listed in Table VI on the following page. Test 3C was given in two installments. In that case the date of the first installment appears, the total score, and the total time taken to complete the entire test.

Process	Date	Test	Score	Time
Addition	2/10/36	3A	100	4:50
	2/13/36	3B	100	5:12
	2/19/36	3C	90	24:00
	2/20/36	3C (short form)	87	11:00
Subtraction	2/25/36	4A	100	5:02
	2/26/36	4B	100	5:26
	2/28/36	4C (short form)	95	7:45
Multiplication	2/29/36	5A	97	4:05
	2/10/36	5B (short form)	92	20:40
Division	2/12/36	6A	76	20:10

vision before. The various step difficulties were introduced step by step. They were illustrated and explained to prevent recurrence of old errors.

Final Test Results. At the conclusion of the remedial instruction, the Wilson Inventory and Diagnostic Tests were administered. The results of the final tests are listed in Table VI on the following page. Test 30 was given in two installments. In that case the date of the first installment appears, the total score, and the total time taken to complete the entire test.

Summary of Case III. About fifty-three testing periods were carried on
 consecutive days over a period of five months. These consecutive periods
 lasted on the average about twenty-minutes each. They were divided among
 the processes approximately as follows: addition 22 periods; subtraction
 12 periods; multiplication 11 periods; and division 8 periods.

TABLE VI

Final Results of Case III on the Wilson Inventory
 and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	3/18/35	3A	100	4:00
	3/18/35	3B	100	5:15
	3/15/35	3C	99	24:50
	3/20/35	3P (short form)	88	11:20
Subtraction	4/20/35	4A	100	3:22
	4/24/35	4B	100	5:20
	4/23/35	4P (short form)	96	7:45
Multiplication	5/10/35	5A	99	8:06
	5/16/35	5P (short form)	92	24:40
Division	6/13/35	6P ₂	76	25:15

TABLE VI

Final Results of Game III on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	3/18/55	3A	100	6:00
	3/18/55	3B	100	6:15
	3/18/55	3C	95	6:30
	3/18/55	3D (short form)	85	11:30
Subtraction	4/20/55	4A	100	7:15
	4/20/55	4B	100	8:30
	4/20/55	4C (short form)	95	7:45
Multiplication	5/10/55	5A	95	8:05
	5/10/55	5B (short form)	95	14:40
Division	5/13/55	5B	75	20:15

Summary of Case III. About fifty-three teaching periods were devoted to corrective work over a period of five-months. These corrective periods lasted on the average about twenty-minutes each. They were divided among the processes approximately as follows: addition 22 periods; subtraction 12 periods; multiplication 11 periods; and division 8 periods.

The final results in Table VI show that the remedial instruction produced distinctly favorable results; gain in scores and decrease in time on the various tests.

He completed the requirements satisfactorily each year and, therefore, was promoted year by year. At present his work in all the other subjects is satisfactory, but arithmetic is where he is deficient. Arithmetic is the only subject that he is failing.

His teacher reports that his arithmetic is low and illustrates a great deal. She thinks that he has the ability to do much better work in the subject.

John has a good attendance record over a period of these five years.

Personality of Child. John is a very cooperative child. He is polite, pleasant, and neat in general appearance. He feels sure of himself, and volunteers to answer questions frequently. During the instructional period he seemed to be confident in his results, and worked rather freely.

He pays divided attention and follows directions carefully. At times he is a bit aggressive, but not talkative. He has a schedule of daily activities outlined, which he follows and enjoys.

John is a very industrious boy. In class he behaves well. He is obedient, and enjoys outdoor games.

Summary of Case III. About fifty-three teaching periods were devoted to

corrective work over a period of five months. These corrective periods lasted on the average about twenty-minutes each. They were divided among the processes approximately as follows: addition 32 periods; subtraction 12 periods; multiplication 12 periods; and division 8 periods.

The final results in Table VI show that the remedial instruction produced distinctly favorable results; gain in scores and decrease in time on the various tests.

CASE IV

At the time this study was made Dann was in the fifth grade. He was born in June, 1924. His intelligence quotient was found to be 90 according to the results of the Otis Group Intelligence Test.

Dann was referred for study by the principal of the Shurtleff School because the teacher reported that he was doing poor in arithmetic.

School History. Dann attended the same school for the past five years.

He completed the requirements satisfactorily each year and, therefore, was promoted year by year. At present his work in all the other subjects is satisfactory, but arithmetic in which he is deficient. Arithmetic is the only subject that he is failing.

His teacher reports that his arithmetic is low and fluctuates a great deal. She thinks that he has the ability to do much better work in the subject.

Dann has a good attendance record over a period of these five years.

Personality of Child. Dann is a very cooperative child. He is polite, pleasant, and neat in general appearance. He feels sure of himself, and volunteers to answer questions frequently. During the instructional periods, he seemed to be confident in his results, and worked rather freely.

He pays strict attention and follows directions carefully. At times he is a bit aggressive, but not talkative. He has a schedule of daily activities outlined, which he follows and enjoys.

Dann is a very industrious boy. In class he behaves well. He is sociable, and enjoys outdoor games.

CASE IV

At the time this study was made Mann was in the fifth grade. He was born in June, 1934. His intelligence quotient was found to be 98 according to the results of the Otis Group Intelligence Test.

Mann was referred for study by the principal of the Hamlet School because the teacher reported that he was doing poor in arithmetic.

School History. Mann attended the same school for the past five years. He completed the requirements satisfactorily each year and, therefore, was promoted year by year. At present his work in all the other subjects is satisfactory, but arithmetic in which he is deficient. Arithmetic is the only subject that he is failing.

His teacher reports that his arithmetic is low and illustrates a great deal. She thinks that he has the ability to do much better work in the subject.

Mann has a good attendance record over a period of these five years.

Personality of Child. Mann is a very cooperative child. He is polite, pleasant, and neat in general appearance. He feels sure of himself, and volunteers to answer questions frequently. During the instructional period, he seemed to be confident in his teacher, and worked rather freely.

He gave strict attention and follows directions carefully. At times

he is a bit aggressive, but not talkative. He has a schedule of daily

activities outlined, which he follows and enjoys.

Mann is a very industrious boy. In class he behaves well. He is sociable, and enjoys outdoor games.

Home Conditions. There are two children in the family, a boy and a girl. Dann's sister is older than he is. She is in the ninth-grade, and attending the same school. Frequently she helps Dann with his school work.

The father is in the upholstery business. He has a small place of his own. The mother's time is devoted to the caring of the home and children.

The family owns and lives in a home in the vicinity of the school. General observations indicate that the home conditions are satisfactory, and that the family lives reasonably comfortable.

Results of Tests in Arithmetic. Table VII gives Dann's initial results on the Wilson Inventory and Diagnostic Test. The time to complete such tests is recorded in every case. Several tests were not administered because of the pressure of time. The tests that were administered, were believed to be sufficient for purposes of diagnosis.

The time in every case was recorded in minutes and seconds, for example, 3:10 for test 3A reads, 3 minutes and 10 seconds. The date the test was administered is also listed, with the key of the test and score obtained on the test.

* Test 3A is an abbreviated form of the 3B test. It contains twenty long division examples, two from each group of the 3B test. Permission was received from the author.

Home Conditions. There are two children in the family, a boy and a girl. Darn's sister is older than he is. She is in the ninth grade, and attends the same school. Presently she helps Darn with his school work. The father is in the upholstery business. He has a small place of his own. The mother's time is devoted to the caring of the home and children. The family owns and lives in a home in the vicinity of the school. General observations indicate that the home conditions are satisfactory, and that the family lives reasonably comfortable.

Results of Tests in Arithmetic. Table VII gives Darn's initial results on the Wilson Inventory and Diagnostic Test. The time to complete each test is recorded in every case. Several tests were not administered because of the pressure of time. The tests that were administered, were believed to be sufficient for purposes of diagnosis. The time in every case was recorded in minutes and seconds, for example, 8:10 for test 3A reads, 3 minutes and 10 seconds. The date the test was administered is also listed, with the key of the test and score obtained on the test.

Analysis of Difficulties. A diagnosis of Sam's work revealed errors and inefficient habits of work. It was based upon the results of the various tests. The following tabulation is a list of the errors and the inefficient habits of work.

TABLE VII

Initial Results of Case IV on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	2/25/35	3A	100	3:10
	2/25/35	3B	100	3:55
	2/25/35	3P (short form)	88	10:30
Subtraction	2/25/35	4P (short form)	96	14:02
Multiplication	2/26/35	5P (short form)	72	29:06
Division	2/28/35	6A	91	19:06
	2/29/35	6P2*	55	41:17

* Test 6P2* is an abbreviated form of the 6P2 test. It contains twenty long division examples, two from each group of the 6P2 test. Permission was received from the author.

TABLE VII

Initial Results of Case IV on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	2/25/55	24	100	3:10
	2/25/55	28	100	3:35
	2/25/55	32 (short form)	98	10:30
Subtraction	2/25/55	42 (short form)	98	14:03
Multiplication	2/25/55	32 (short form)	75	23:35
Division	2/25/55	44	91	17:08
	2/25/55	52*	95	41:17

* Test 52* is an abbreviated form of the 52 test. It contains twenty long division examples, two from each group of the 52 test. Permission was received from the author.

Analysis of Difficulties. A diagnosis of Dann's work revealed errors and inefficient habits of work. It was based upon the results of the various tests. The following tabulation is a list of the errors and the inefficient habits of work:

Addition

1. Split numbers
2. Grouped numbers
3. Added carried number irregularly

Subtraction

1. Used cumbersome method

Multiplication

1. Errors in primary combinations
2. Zero difficulty in single combinations
3. Errors in addition

Division

1. Errors in division combinations
2. Errors in subtraction
3. Errors in multiplication
4. Used remainder larger than divisor
5. Confused by dollar sign and decimal point

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests will follow. They were taken from the various tests, and appear in the same form.

Analysis of Difficulties. A diagnosis of Barn's work revealed errors and inefficient habits of work. It was based upon the results of the various tests. The following tabulation is a list of the errors and the inefficient habits of work:

Addition

- 1. Split numbers
- 2. Grouped numbers
- 3. Added carried number irregularly

Subtraction

- 1. Used cumbersome method

Multiplication

- 1. Errors in primary computations
- 2. Zero difficulty in single computations
- 3. Errors in addition

Division

- 1. Errors in division computations
- 2. Errors in subtraction
- 3. Errors in multiplication
- 4. Used remainder larger than divisor
- 5. Confused by dollar sign and decimal point

Specific Illustrations of Difficulties. The specific difficulties that

were revealed from an analysis of the various tests will follow. They were taken from the various tests, and appear in the same form.

Difficulties in Addition

1. Split numbers

Example:

$$\begin{array}{r} 8 \\ 9 \\ \hline 17 \end{array}$$

In working the example on the left, he borrowed 1 from the 8, and added it to the 9 and made it 10. He then added the 10 and 7 mentally and wrote down the answer. He followed such a procedure only on certain combinations.

2. Grouped numbers

Example:

$$\begin{array}{r} 2 \\ 6* \\ 4* \\ 2 \\ 4 \\ \hline 18 \end{array}$$

In adding columns, this child group any two number which equalled to 10. In this example, he grouped the 6 and 4 as 10, and proceeded to add the other numbers in the column.

3. Added carried number irregularly

Example:

$$\begin{array}{r} \$.17 \\ 5.37 \\ 2.57 \\ 6.75 \\ 4.86 \\ \hline \$19.52 \end{array}$$

In working this example, he obtained the answer for the first column, and instead of adding the carried to the first digit in the second column, he added it to the second digit in that column. He did this, because 3 and 7 added together make 10.

When questioned about the procedure in adding carried numbers, he said that sometimes he added first, sometimes within the column, and sometimes at the very end.

Difficulties in Subtraction

1. Used cumbersome method

Example:

$$\begin{array}{r} 14 \\ 8 \\ \hline 6 \end{array}$$

In working this example, he said, "2 added to 8 makes 10; 2 + 4 is 6, and he wrote 6 for the answer. Frequently, he worked other examples in the same manner. This procedure led to difficulty in examples that contained two, three or more digits in both, the subtrahend and minuend.

Difficulties in addition

1. Split numbers

Example:

In working this example on the left, he borrowed 1 from the 8, and added it to the 9 and made it 10. He then added the 10 and 7 mentally and wrote down the answer. He followed with a procedure only on certain combinations.

$$\begin{array}{r} 9 \\ 7 \\ \hline 16 \end{array}$$

2. Grouped numbers

Example:

In adding columns, this child group any two number which equalled 10. In this example, he grouped the 8 and 2 as 10, and proceeded to add the other numbers in the column.

$$\begin{array}{r} 8 \\ 2 \\ 4 \\ 2 \\ \hline 16 \end{array}$$

3. Added carried number irregularly

Example:

In working this example, he obtained the answer for the first column, and instead of adding the carried to the first digit in the second column, he added it to the second digit in that column. He did this because 2 and 7 added together make 10. When questioned about the procedure in adding carried numbers, he said that sometimes he added first, sometimes within the column, and sometimes at the very end.

$$\begin{array}{r} 9 \\ 7 \\ 2 \\ 2 \\ 2 \\ \hline 16 \end{array}$$

Difficulties in subtraction

1. Used cumbersome method

Example:

In working this example, he said, "2 added to 8 makes 10; 8 + 4 is 12, and he wrote 8 for the answer. Apparently, he worked other examples in the same manner. This procedure led to difficulty in examples that contained two, three or more digits in both the subtrahend and minuend.

$$\begin{array}{r} 12 \\ 8 \\ \hline 2 \end{array}$$

Difficulties in Multiplication

1. Errors in primary combinations

Example 1:

$$\begin{array}{r} 8 \\ 6 \\ \hline 54 \end{array}$$

He worked this example as shown.

Example 2:

$$\begin{array}{r} 445 \\ 308 \\ \hline 3665 \end{array}$$

The example is shown in part. In working it, he multiplied 8 times 5 as 45.

2. Zero difficulty in single combinations

Examples:

$$\begin{array}{r} 7 \quad 0 \quad 0 \quad 4 \quad 0 \\ 0 \quad 8 \quad 1 \quad 0 \quad 5 \\ \hline 7 \quad 8 \quad 1 \quad 4 \quad 5 \end{array}$$

3. Errors in addition

Example :

$$\begin{array}{r} 7081 \\ 509 \\ 63729 \\ 354050 \\ \hline 3604329 \end{array}$$

In working this example, he obtained the correct answers for the partial products, but in adding the third column he made a mistake.

Difficulties in Division

1. Errors in division

The following are some of the division combinations missed:

$$7 \overline{) 14}$$

$$6 \overline{) 48}$$

$$7 \overline{) 56}$$

$$8 \overline{) 24}$$

$$9 \overline{) 63}$$

Difficulties in Multiplication

1. Errors in primary combinations

Example 1:

$$\begin{array}{r} 8 \\ 8 \\ \hline 64 \end{array}$$

He worked this example as shown.

Example 2:

$$\begin{array}{r} 440 \\ 800 \\ \hline 3520 \end{array}$$

The example is shown in part. In working it, he multiplied 8 times 8 as 48.

2. Zero difficulty in single combinations

Examples:

$$\begin{array}{r} 7 \\ 0 \\ \hline 7 \end{array} \quad \begin{array}{r} 0 \\ 0 \\ \hline 0 \end{array} \quad \begin{array}{r} 0 \\ 1 \\ \hline 1 \end{array} \quad \begin{array}{r} 4 \\ 0 \\ \hline 4 \end{array} \quad \begin{array}{r} 0 \\ 0 \\ \hline 0 \end{array}$$

3. Errors in addition

Example:

$$\begin{array}{r} 7001 \\ 800 \\ 8700 \\ \hline 22400 \end{array}$$

In working this example, he obtained the correct answer for the partial products, but in adding the third column he made a mistake.

Difficulties in Division

1. Errors in division

The following are some of the division combinations missed:

$$\begin{array}{r} 7 \\ 14 \\ \hline 2 \end{array} \quad \begin{array}{r} 7 \\ 21 \\ \hline 3 \end{array} \quad \begin{array}{r} 8 \\ 24 \\ \hline 3 \end{array} \quad \begin{array}{r} 4 \\ 12 \\ \hline 3 \end{array}$$

2. Errors in subtraction

Example:

$$\begin{array}{r}
 32 \quad 32/61 \\
 61 \overline{) 1974} \\
 \underline{183} \\
 15^*4 \\
 \underline{122} \\
 32
 \end{array}$$

In working this example, he subtracted incorrectly in the first subtraction. The position is indicated by the asterisk.

3. Errors in multiplication

Example:

$$\begin{array}{r}
 91 \quad 40/54 \\
 54 \overline{) 4914} \\
 \underline{482} \\
 94 \\
 \underline{54} \\
 40
 \end{array}$$

In working this example, he multiplied 54 by 9, and obtained a product of 482.

4. Used remainder larger than divisor

Example:

$$\begin{array}{r}
 21 \quad 50/63 \\
 63 \overline{) 2394} \\
 \underline{126} \\
 *113 \\
 \underline{63} \\
 50
 \end{array}$$

In working this example, he used a remainder larger than the divisor.

5. Confused by dollar sign and decimal point

Example:

$$\begin{array}{r}
 \$ \quad 7 \\
 45 \overline{) \$31.50} \\
 \underline{31.5} \\
 0
 \end{array}$$

This pupil worked the example as shown.

General Plan of Remedial Instruction. The purpose of the remedial instruction was to remove the specific difficulties and to engender more efficient habits of work. During instructional periods, the disadvantages of faulty procedures that arose, were explained and illustrated by more

2. Error in subtraction

Example:

$$\begin{array}{r} 22/61 \\ 1874 \\ 185 \\ \hline 174 \\ 134 \\ \hline 134 \\ 22 \end{array}$$

In working this example, he subtracted incorrectly in the first subtraction. The position is indicated by the asterisk.

3. Error in multiplication

Example:

$$\begin{array}{r} 20/24 \\ 4814 \\ 482 \\ \hline 9628 \\ 84 \\ \hline 9712 \end{array}$$

In working this example, he multiplied 24 by 2, and obtained a product of 48.

4. Used remainder larger than divisor

Example:

$$\begin{array}{r} 20/22 \\ 5594 \\ 1118 \\ \hline 1118 \\ 1118 \\ \hline 1118 \\ 22 \end{array}$$

In working this example, he used a remainder larger than the divisor.

5. Confused by dollar sign and decimal point

Example:

$$\begin{array}{r} 2.7 \\ 25/1.35 \\ 1.35 \\ \hline 1.35 \\ 0 \end{array}$$

This pupil worked the example as shown.

General Plan of Remedial Instruction. The purpose of the remedial instruction was to remove the specific difficulties and to engender more efficient habits of work. During instructional periods, the disadvantages of faulty procedures that arose, were explained and illustrated by more

efficient habits of work. He was encouraged to use the new procedures until they replaced the older ones.

Arman was allowed to work the examples in just the way he would normally work them. The purpose was, to arrive at a more complete diagnosis, and to avoid later remedial instruction. Such a plan permitted further discovery of difficulties. He was also requested to compute examples aloud so that his method of thinking thru the example might be checked. Usually such information was supplemented by further questioning.

During the remedial program the 100% Drill Services were employed to furnish the practice exercises. Informal examinations were administered from time to time to check for mastery. Dann, as was stated before, worked in cooperation with Arman. They enjoyed working together very much.

Detailed Account of Remedial Instruction. The following paragraphs describe the methods that were employed to correct the specific difficulties that the child encountered in working with the processes. Reference to the list of the detailed description of habits given in the earlier part of this case study will enable the reader to identify the habits being considered at any time.

Addition

To overcome the habit of splitting numbers, he studied the primary and related facts from individual flash cards until he could respond to them automatically.

To overcome the habit of grouping numbers and the habit of adding the carried number irregularly, he was shown by illustration the correct procedure in column addition. From time to time, he was requested to do aloud

efficient habits of work. He was encouraged to use the new procedures

until they regained the older ones.

Arman was allowed to work the examples in just the way he would normally

work them. The purpose was to arrive at a more complete diagnosis, and

to avoid later remedial instruction. Such a plan permitted further dis-

covery of difficulties. He was also requested to compare examples along

so that his method of thinking about the example might be checked. Usually

such information was supplemented by further questioning.

During the remedial program the 1904 Drill Services were assigned to

perform the practice exercises. Informal examinations were administered

from time to time to check for mastery. Sam, as was stated before, worked

in cooperation with Arman. They enjoyed working together very much.

Detailed Account of Remedial Instruction. The following paragraphs de-

scribe the methods that were employed to correct the specific difficulties

that the child encountered in working with the processes. Reference to

the list of the detailed description of habits given in the earlier part

of this case study will enable the reader to identify the habits being cor-

rected at any time.

addition

To overcome the habit of splitting numbers, he studied the primary

and related facts from individual flash cards until he could respond to

them automatically.

To overcome the habit of grouping numbers and the habit of adding the

carried number incorrectly, he was shown by illustration the correct pro-

cedure in column addition. From time to time, he was requested to do along

examples involving column addition. The reason being, to constantly check the method that he employed, and to encourage the use of the better procedure until the older habits fell into disuse.

Subtraction

To overcome the cumbersome method that he employed in subtraction, he studied the primary facts from individual flash cards, just as he did in the case of the addition facts.

Multiplication

To overcome the errors in primary combinations, Dann studied them from individual flash cards until he mastered them. He studied the one-hundred facts.

Since he studied all the primary facts and mastered them, the single combinations with a zero in them no longer troubled him.

To overcome the errors in addition, they were called to his attention whenever they appeared. In such cases he was requested to make the necessary corrections. That plan was followed throughout the entire corrective program.

Division

Because of the lack of time, division was not treated as intensively as the other processes, but, nevertheless the various difficulties were taken up.

The diagnosis revealed that Dann was not entirely ignorant of this process, but did not understand it thoroughly. It was therefore decided to reteach the process of long division step by step. First, the simple ex-

examining column addition. The reason being, to constantly check the method that he employed, and to encourage the use of the better procedure until the older habits fall into disuse.

Subtraction

To overcome the cumbersome method that he employed in subtraction, he studied the primary facts from individual flash cards, just as he did in the case of the addition facts.

Multiplication

To overcome the error in primary combinations, Dunn studied them from individual flash cards until he mastered them. He studied the one-hundred facts.

Since he studied all the primary facts and mastered them, the simple combinations with a zero in them no longer troubled him. To overcome the errors in addition, they were called to his attention whenever they appeared. In such cases he was requested to make the necessary corrections. That plan was followed throughout the entire corrective program.

Division

Because of the lack of time, division was not treated as intensively as the other processes, but nevertheless the various difficulties were taken up. The diagnosis revealed that Dunn was not entirely ignorant of this process, but did not understand it thoroughly. It was therefore decided to review the process of long division step by step. First the steps ex-

amples were taken up, and later they were followed by the more difficult ones. Each one was illustrated and explained to insure understanding. Following this, some drill was given to engender the correct habits of work.

Final Test Results. At the conclusion of the remedial instruction, the Wilson Inventory and Diagnostic Test were administered. The results of the final tests are listed in Table VIII on the following page.

Category	Date	Test	Score	Time
Division	3/15/35	11	100	3:35
	3/16/35	23	75	4:01
	3/18/35	30	88	3:40
	3/20/35	37 (short form)	92	10:00
Subtraction	4/23/35	47 (short form)	100	3:42
Multiplication	4/24/35	57 (short form)	88	17:00
Division	4/25/35	62	88	21:00

copies were taken up, and later they were followed by the more difficult ones. Each one was illustrated and explained to insure understanding. Following this, some drill was given to engender the correct habits of work.

Final Test Results. At the conclusion of the remedial instruction, the Wilson Inventory and Diagnostic Test were administered. The results of the final tests are listed in Table VIII on the following page.

TABLE VIII

Final Results of Case IV on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	3/18/35	3A	100	3:35
	5/18/35	3B	99	4:01
	3/15/35	3C	99+	25:40
	3/20/35	3P (short form)	92	10:02
Subtraction	4/26/35	4P (short form)	100	8:42
Multiplication	5/16/35	5P (short form)	88	17:02
Division	6/13/35	6P ₂	85	31:04

TABLE VIII

Final Results of Case IV on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	2/18/35	3A	100	3:35
	2/18/35	3B	99	4:01
	2/18/35	3C	99+	35:40
	2/20/35	3P (short form)	99	10:08
Subtraction	2/20/35	4P (short form)	100	8:45
Multiplication	2/18/35	3P (short form)	99	17:08
Division	2/12/35	85%	99	21:04

Summary of Case IV. About forty-five teaching periods were devoted to corrective work over a period of three and one-half months. These corrective periods lasted on the average about twenty minutes each. They were divided among the processes approximately as follows: addition 14 periods; subtraction 12 periods; multiplication 11 periods; and division 8 periods.

The final results in Table VIII show that the remedial instruction produced favorable results; gain in scores and decrease in time. In two instances losses are reported. A slight increase in time for test 3A, and a slight increase in time for test 3B and decrease in score. However, these losses can be considered negligible because the change in score and time is very small. They might be attributed to causes other than lack of understanding of the process.

Summary of Data IV. About forty-five teaching periods were devoted to

corrective work over a period of three and one-half months. These cor-

rective periods lasted on the average about twenty minutes each. They

were divided among the programs approximately as follows: addition 14

periods; subtraction 13 periods; multiplication 11 periods; and division

8 periods.

The final results in Table VII show that the remedial instruction

produced favorable results; gain in scores and decrease in time. In two

instances losses are reported. A slight increase in time for test 2A,

and a slight increase in time for test 2B and decrease in score. How-

ever, these losses are considered negligible because the change in score

and time is very small. They might be attributed to causes other than

lack of understanding of the process.

CASE V

At the time this study was made Dominic was in the sixth grade. He was born in March, 1923. His intelligence quotient was found to be 96 according to the results of the Otis Group Intelligence Test. (1)

This boy was referred for study by the Judge Baker Guidance Center because his work in arithmetic was very poor, and he was referred for study by the court to the Judge Baker Guidance Center because of excessive truancy. (2)

School History. (3) Dominic's school records could not be obtained from the authorities. However, the boy reported that he began school at the age of four, and that the work in the first few grades was difficult.

During this study, he was later transferred from the public school to a parochial.

Personality of Child. He is a very sad looking boy and quite attractive. He appears to be excessively irritable. In test periods he seems quiet and earnest.

This boy maintains that the sole cause of his dissatisfaction for school is his lack of understanding numbers. Dominic is an educational problem on account of his difficulty with arithmetic. He is also a personality difficulty. He behaves poorly at home and at school. He is unhappy on account of his home situation, feeling that he is misunderstood and unappreciated. His school situation makes him depressive. (4)

Home Conditions. (5) Dominic is the second of four living children. He is at home with his parents, both are quite illiterate. The father is an un-

(1) Case study No. 9114. Judge Baker Guidance Center

(2) Ibid.

(3) Ibid.

(4) Ibid.

(5) Ibid.

At the time this study was made Dominic was in the sixth grade. He was born in March, 1933. His intelligence quotient was found to be 88.

According to the results of the Otis Group Intelligence Test, (1)

This boy was referred for study by the Judge Baker Guidance Center

because his work in arithmetic was very poor, and he was referred for study by the court to the Judge Baker Guidance Center because of excessive

triancy. (2)

School History. (3) Dominic's school records could not be obtained from

the authorities. However, the boy reported that he began school at the

age of four, and that the work in the first few grades was difficult.

During this study, he was later transferred from the public school

to a parochial.

Personality of Child. He is a very well looking boy and quite attractive.

He appears to be excessively irritable. In past periods he seems quiet

and earnest.

This boy maintains that the cause of his dissatisfaction for

school is his lack of understanding numbers. Dominic is an educational

problem on account of his difficulty with arithmetic. He is also a person-

ally difficult. He behaves poorly at home and at school. He is unhappy

on account of his home situation, feeling that he is misunderstood and un-

appreciated. His school situation makes him depressive. (4)

Home Conditions. (5) Dominic is the second of four living children. He is

at home with his parents, both are quite illiterate. The father is an un-

usually home loving man of good habits. He has very few friends and indulges in almost no form of recreation. He spends most of his time at home doing nothing.

The family lives in an apartment in a poor tenement house, which is very well kept. The mother is evidently a good housekeeper.

Dominic has been the mother's particular object of concern. The younger brother has been the father's favorite. Dominic has reacted to this by considerable show of impudence to his father. He is highly critical of him, and makes much of his father's ignorance and social inaptitude. He feels keenly that his father does not love him.

Results of Tests in Arithmetic. Table IX gives Dominic's initial results on the Wilson Inventory and Diagnostic Test. Tests 3P and 4P were given in two installments each. Test 5A was given in three installments. In such cases the date of the first installment appears, the total score, and the total time taken to complete the entire test. The time reads in minutes and seconds, for example, 2:57 reads 2 minutes and 57 seconds.

usually home loving man of good habits. He has very few friends and in-
duces in almost no form of recreation. He spends most of his time at
home doing nothing.

The family lives in an apartment in a poor tenement house, which is
very well kept. The mother is evidently a good housekeeper.

Domino has been the mother's particular object of concern. The young-
er brother has been the father's favorite. Domino has seemed to take by
considerable show of ignorance to his father. He is highly critical of him,
and makes much of his father's ignorance and social ineptitude. He feels
bitterly that his father does not love him.

Results of Tests in Arithmetic. Table IX gives Domino's initial results

on the Wilson Inventory and Arithmetic Test. Tests 25 and 26 were given
in two installments each. Test 24 was given in three installments. In such
cases the date of the first installment appears, the total score, and the
total time taken to complete the entire test. The time reads in minutes
and seconds, for example, 2:57 reads 2 minutes and 57 seconds.

TABLE IX

Initial Results of Case V on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	11/27/34	3A	99	2:57
	11/27/34	3B	96	6:06
	11/27/34	3C	99	25:30
	11/27/34	3P (long form)	89	33:00
Subtraction	11/28/34	4P (long form)	86	29:45
Multiplication	11/28/34	5P (long form)	53	57:20
Division	12/ 2/34	6P ₂ *

* It took Dominic 36 minutes to work on 14 examples in the 6P₂ test, six of which were incorrect. From the way he worked, it was evident that he had no system and was generally confused.

TABLE IX
Initial Results of Case V on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	11/27/34	2A	93	8:07
	11/27/34	2B	93	8:08
	11/27/34	2C	93	8:20
	11/27/34	2F (long form)	93	12:00
Subtraction	11/28/34	4P (long form)	88	12:43
	11/28/34	5P (long form)	83	12:50
Multiplication	12/2/34	6P*

* It took Dominic 36 minutes to work on 14 examples in the 6P test, six of which were incorrect. From the way he worked, it was evident that he had no system and was generally confused.

Analysis of Difficulties. A diagnosis based upon his work on the various test, revealed errors and inefficient habits of work. The following tabulation gives a list of the various errors and inefficient habits or work discovered:

Addition

1. Errors in primary combinations
2. Errors in upper decade combinations
3. Counting
4. Wrote number to be carried
5. Grouped numbers

Subtraction

1. Errors in primary combinations
2. Last subtraction a zero brought down
3. Counting
4. Deducted from minuend when borrowing was not necessary
5. Did not allow for having borrowed
6. Added instead of subtracting

Multiplication

1. Errors in primary combinations
2. Errors in adding the carried number
3. Errors in addition
4. Errors due to zero in multiplier
5. Forgot to carry
6. Counted to get multiplication combination
7. Wrote the carried number

Analysis of Difficulties. A diagnosis based upon his work on the various test, revealed errors and inefficient habits of work. The following tabulation gives a list of the various errors and inefficient habits of work discovered:

Addition

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2. Errors in upper decade combinations
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5. Grouped numbers

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4. Deducted from minuend when borrowing was not necessary
5. Did not allow for having borrowed
6. Added instead of subtracting

Multiplication

1. Errors in primary combinations
2. Errors in adding the carried number
3. Errors in addition
4. Errors due to zero in multiplier
5. Forgot to carry
6. Counted to get multiplication combination
7. Wrote the carried number

8. Misplace^d and omitted decimal point (U.S. Money)
9. Used wrong process

Division

1. General confusion

Specific Illustrations of Difficulties. The specific difficulties that were revealed from an analysis of the various tests are illustrated below. They are shown in the same form as they appear in the original tests.

Difficulties in Addition

1. Errors in primary combinations

The following combinations were missed:

$$\begin{array}{r} 7 \\ 6 \\ \hline 15 \end{array} \quad \begin{array}{r} 2 \\ 2 \\ \hline 5 \end{array} \quad \begin{array}{r} 9 \\ 3 \\ \hline 13 \end{array} \quad \begin{array}{r} 7 \\ 6 \\ \hline 15 \end{array} \quad \begin{array}{r} 7 \\ 8 \\ \hline 16 \end{array} \quad \begin{array}{r} 7 \\ 5 \\ \hline 11 \end{array}$$

2. Errors in upper decade combinations

The following upper decade combinations were missed:

$$31 + 3 = 16$$

$$23 + 4 = 28$$

$$27 + 5 = 30$$

3. Counting

Example:

$$\begin{array}{r} 9 \\ 8 \\ \hline 17 \end{array}$$

This pupil did not know all his combinations, therefore, resorted to counting. In working this example, he said, "8 and 9 are - 9, 10, 11, 12, 13, 14, 15, 16, 17," tapping each time with his finger when he said a new number.

8. Missions and omitted decimal points (U.S. money)

9. Used wrong process

Division

1. General conclusion

Specific illustrations of difficulties. The specific difficulties that

were revealed from an analysis of the various tests are illustrated below.

They are shown in the same form as they appear in the original tests.

Difficulties in Addition

1. Errors in primary combinations

The following combinations were missed:

7	2	9	7	7
6	3	8	6	6
18	5	17	13	11

2. Errors in upper decade combinations

The following upper decade combinations were missed:

$$31 + 5 = 16$$

$$23 + 4 = 28$$

$$27 + 5 = 30$$

3. Counting

Example:

This pupil did not know all the combinations, therefore, resorted to counting. In working this example, he said "8 and 9 are - 8, 10, 11, 12, 13, 14, 15, 16, 17," tapping each time with his finger when he said a new number.

4. Wrote number to be carried

Example:

$\begin{array}{r} 78 \\ 64 \\ 97 \\ 9 \\ 78 \\ \hline 326 \end{array}$	<p>In working this example, he obtained the sum for the first column and instead of adding the carried first to the next column, he wrote it in the gap and then proceeded to add the column.</p>
------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

5. Grouped numbers

Example:

$\begin{array}{r} 87 \\ 55 \\ 91 \\ 69 \\ 54 \\ \hline 356 \end{array}$	<p>In the first column of this example, he grouped 1 and 9 as 10, using it as a single addend, and then proceeded with the rest of the example.</p>
-------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------

Difficulties in Subtraction

1. Errors in primary combinations

Examples:

$\begin{array}{r} 13 \\ 7 \\ \hline 09 \end{array}$	$\begin{array}{r} 1759 \\ 843 \\ \hline 0654 \end{array}$
-----------------------------------------------------	-----------------------------------------------------------

2. Last subtraction a zero brought down

Example 1:

$\begin{array}{r} 96 \\ 91 \\ \hline 05 \end{array}$	<p>He worked the example as shown.</p>
------------------------------------------------------	----------------------------------------

Example 2:

$\begin{array}{r} 125 \\ 54 \\ \hline 071 \end{array}$	<p>He worked this example as shown.</p>
--------------------------------------------------------	-----------------------------------------

4. Wrote number to be carried

Example:

78
84
97
8
78

235

In working this example, he obtained the sum for the first column and instead of adding the carried first to the next column, he wrote it in the gap and then proceeded to add the column.

5. Crossed numbers

Example:

87
33
91
82
34

335

In the first column of this example, he crossed 1 and 3 as 10, making it a single addend, and then proceeded with the rest of the example.

Difficulties in Subtraction

1. Wrote in primary combinations

Example:

18
7

25
1733
823

9554

2. Last subtraction zero brought down

Example 1:

78
31

47

He worked the example as shown.

Example 2:

123
54

177

He worked this example as shown.

3. Counting

Example:

$$\begin{array}{r} 829 \\ 53 \\ \hline 776 \end{array}$$

In working this example, he deducted three fingers from nine fingers, and put down what was left. In the second column, he deducted two fingers beginning with the thumb of the left hand and went back again to deduct three more, a total of five fingers, and then put down what was left as the answer. He obtained the answers for most of the examples in a similar way.

4. Deducted from minuend when borrowing was not necessary

Example:

$$\begin{array}{r} 230 \\ 116 \\ \hline 014 \end{array}$$

He worked this example as shown.

5. Did not allow for having borrowed

Example:

$$\begin{array}{r} \$10.00 \\ 6.25 \\ \hline \$13.75 \end{array}$$

In the last column, he brought down the 1.

Difficulties in Multiplication

1. Errors in primary combinations

Example 1:

$$\begin{array}{r} 8 \\ 6 \\ \hline 52 \end{array}$$

Example 2:

$$\begin{array}{r} 91 \\ 7 \\ \hline 727 \end{array}$$

In multiplying 7 times 9, he obtained 72.

Example 3:

$$\begin{array}{r} 68 \\ 6 \\ \hline 423 \end{array}$$

In working this example, he obtained 63 as the product of 6 and 8.

Counting

Example:

$$\begin{array}{r} 23 \\ 23 \\ \hline 46 \end{array}$$

In working this example, he deducted three fingers from nine fingers, and put down what was left. In the second column, he deducted two fingers beginning with the thumb of the left hand and went back again to deduct three more, a total of five fingers, and then put down what was left as the answer. He obtained the answers for most of the examples in a similar way.

1. Borrowed from nine when borrowing was not necessary

Example:

$$\begin{array}{r} 230 \\ 115 \\ \hline 345 \end{array}$$

He worked this example as above.

2. Did not allow for having borrowed

Example:

$$\begin{array}{r} 40.00 \\ 3.33 \\ \hline 36.67 \end{array}$$

In the last column, he brought down the 1.

Multiplication in Indigian

1. Errors in primary combinations

Example 1:

$$\begin{array}{r} 2 \\ 2 \\ \hline 4 \end{array}$$

Example 2:

$$\begin{array}{r} 21 \\ 7 \\ \hline 147 \end{array}$$

In multiplying 7 times 21, he obtained 147.

Example 3:

$$\begin{array}{r} 66 \\ 3 \\ \hline 198 \end{array}$$

In working this example, he obtained 66 as the product of 3 and 22.

2. Errors in adding the carried number

Example:

$$\begin{array}{r} 53 \\ 7 \\ \hline 361 \end{array}$$

In working this example, he made an error in adding the carried number. He worked the example as shown.

3. Errors in addition

Example:

$$\begin{array}{r} 71 \\ 43 \\ \hline 213 \\ 284 \\ \hline 3043 \end{array}$$

In working this example, he obtained the correct partial products, but in adding these partial products, he added 4 and 1 as 4.

4. Errors due to zero in multiplier

Example:

$$\begin{array}{r} 700 \\ 60 \\ \hline 700 \\ 4200 \\ \hline 42700 \end{array}$$

This pupil multiplied by the zero as though it was 1.

5. Forgot to carry

Example:

$$\begin{array}{r} \$2.50 \\ 4 \\ \hline \$8.00 \end{array}$$

In this example, he neglected to add the carried number.

6. Counted to get multiplication combination.

Example:

$$\begin{array}{r} 76 \\ 7 \\ \hline 532 \end{array}$$

This pupil knew tables up to 3 reasonably well, but tables beyond that he did not know. In working this example, he said, "3 times 6 is 18 - 19, 20, 21, 22, 23, 24 - 25, 26, 27, 28, 29, 30 - 31, 32, 33, 34, 35, 36 - 37, 38, 39, 40, 41, 42." He counted by one's in groups of six. He said that he resorted to such frequently in order to obtain the product.

3. Error in adding the carried number

Example:

In working this example, he made an error in adding the carried number. He wrote the example as shown.

$$\begin{array}{r} 25 \\ 7 \\ \hline 32 \end{array}$$

4. Error in addition

Example:

In working this example, he obtained the correct partial products, but in adding these partial products, he added 4 and 1 as 6.

$$\begin{array}{r} 71 \\ 25 \\ \hline 210 \\ 175 \\ \hline 385 \end{array}$$

5. Error due to zero in multiplier

Example:

This pupil multiplied by the zero as though it was 1.

$$\begin{array}{r} 700 \\ 20 \\ \hline 700 \\ 1400 \\ \hline 14700 \end{array}$$

6. Forget to carry

Example:

In this example, he neglected to add the carried number.

$$\begin{array}{r} 22.50 \\ 2 \\ \hline 45.00 \end{array}$$

7. Counted to get multiplication combination

Example:

This pupil knew tables up to 5 reasonably well, but tables beyond that he did not know. In working this example, he said, "5 times 6 is 10 - 12, 10, 11, 12, 13, 14 - 25, 26, 27, 28, 29, 30 - 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42." He counted by one's in groups of six. He said that he resorted to such frequency in order to obtain the product.

$$\begin{array}{r} 48 \\ 7 \\ \hline 336 \end{array}$$

7. Wrote carried number

Example:

$$\begin{array}{r} 97 \\ 4 \\ \hline 368 \end{array}$$

In working this example, he wrote the carried number to one side. He said that he did it frequently.

8. Misplaced and omitted decimal point (U.S. Money)

Example 1:

$$\begin{array}{r} \$7.10 \\ 3 \\ \hline \$2.130 \end{array}$$

In working this example, he obtained the correct product, but misplaced the decimal point.

Example 2:

$$\begin{array}{r} \$8.05 \\ 7 \\ \hline 7235 \end{array}$$

In working this example, he omitted the decimal point and neglected the dollar sign. He also obtained an incorrect product.

9. Used wrong process

Example:

$$\begin{array}{r} 36 \\ 17 \\ \hline 53 \\ 53 \\ \hline 583 \end{array}$$

In working this example, he added instead of multiplying.

General Plan of Remedial Instruction. The purpose of remedial instruction was to remove the various difficulties which Dominic experienced in working with the fundamental processes of arithmetic. The nature of the difficulties revealed from an analysis of the work, indicated a necessity of reteaching the processes.

The first plan was to get on friendly terms with the boy in order to develop an interest in the work and to enlist his cooperation with the view of improvement. After this relationship was established, he was given My Addition Drill Book (1), the plan of which was outlined to him.

7. Wrote carried number

Example:

In working this example, he wrote the carried number as one side. He said that he did it frequently.

$$\begin{array}{r} 37 \\ 4 \\ \hline 38 \end{array}$$

8. Misplaced and omitted decimal point (U.S. Money)

Example 1:

In working this example, he obtained the correct product, but misplaced the decimal point.

$$\begin{array}{r} 27.10 \\ 3 \\ \hline 81.30 \end{array}$$

Example 2:

In working this example, he omitted the decimal point and neglected the dollar sign. He also obtained an incorrect product.

$$\begin{array}{r} 28.08 \\ 7 \\ \hline 196.56 \end{array}$$

9. Used wrong process

Example:

In working this example, he added instead of multiplying.

$$\begin{array}{r} 55 \\ 17 \\ 35 \\ \hline 28 \end{array}$$

General View of Remedial Instruction. The purpose of remedial instruction

was to remove the various difficulties which pupils experienced in working with the fundamental processes of arithmetic. The nature of the difficulties revealed from an analysis of the work, indicated a necessity of reorganizing the processes.

The first plan was to put on friendly terms with the boy in order to develop an interest in the work and to elicit his cooperation with the view of improvement. After this relationship was established, he was given an addition drill book (1), the plan of which was outlined to him.

He studied the first group in that book, and after completing it satisfactorily he was promoted to the second group. Throughout the entire remedial program such a plan was employed. During the instructional periods the correct procedures were illustrated to insure understanding in order to avoid repetition of old errors. Frequently, he was requested to work examples aloud so that his methods of thinking thru such examples might be checked. The work was carefully watched and errors were never allowed to pass unnoted. Whenever errors appeared, he was requested to make the necessary corrections before going on to further work.

From time to time informal examinations were administered to check for mastery. At the conclusion of the remedial instruction in addition, a final test was given. After that, he worked on subtraction.

No corrective work was done in multiplication and division because Dominic had failed to report for further help.

Detailed Account of Remedial Instruction. The following paragraphs describe the methods employed for the purpose of correcting the various difficulties that Dominic encountered in working with addition and subtraction. Reference to the list of the detailed description of the difficulties given in the earlier part of this Case will enable the reader to identify the ones being considered at any time.

Addition

Flash cards were used to overcome the errors in primary and related facts. The facts were studied from individual cards until he could respond to them automatically.

After he mastered the facts within the first group of the 100% Drill

he studied the first group in last book, and after completing it satisfactorily he was promoted to the second group. Throughout the entire remedial program such a plan was employed. During the instructional periods the correct procedures were illustrated to insure understanding in order to avoid repetition of old errors. Presently, he was requested to work examples aimed so that his methods of thinking thru such examples might be checked. The work was carefully watched and errors were never allowed to pass unnoticed. Whenever errors appeared, he was requested to make the necessary corrections before going on to further work.

From time to time informal examinations were administered to check for mastery. At the conclusion of the remedial instruction in addition, a final test was given. After that, he worked on subtraction. No corrective work was done in multiplication and division because Dominic had failed to report for further help.

Detailed Account of Remedial Instruction. The following paragraphs describe the methods employed for the purpose of correcting the various difficulties that Dominic encountered in working with addition and subtraction. Reference to the list of the detailed description of the difficulties given in the earlier part of this case will enable the reader to identify the ones being considered at any time.

Addition

Flash cards were used to overcome the errors in primary and related facts. The facts were stated from individual cards until he could respond to them automatically. After he mastered the facts within the first group of the 100's Drill

Service, he worked the examples within that group. The same procedure was followed for the second group in that book, the third group, etc. Gradually, by such a plan the habit of counting was not resorted to.

To overcome the habit of writing the carried number, he received instructions to add the carried always to the first number in the next column. During the instructional periods he was constantly checked on that habit by requesting him to do certain examples aloud.

To overcome the habit of grouping numbers, the procedure of adding the numbers in a successive order was illustrated. Also the possibilities of errors arising by such a procedure as he employed was explained. During the instructional periods he was encouraged to add the numbers in the successive order. Occasionally he was requested to work aloud for purposes of checking.

Subtraction

Flash cards were employed to overcome the errors in primary combinations.

To overcome subtraction habit 2, it was explained that zero in such cases was not necessary. Following this, practice exercises were assigned and future work carefully watched for the reappearance of this habit. Whenever it reappeared, he was requested to make corrections and reminded again that the zero is not essential in such cases.

To overcome the habit of counting he studied the facts so that he could respond to them automatically.

To overcome subtraction habits 4 and 5, the method of subtracting such examples that caused him difficulty were illustrated. Following this,

Service, he worked the examples within that group. The same procedure was followed for the second group in that book, the third group, etc. (Continued)

1. by and when the habit of counting was not resorted to.

To overcome the habit of writing the carried number, he resorted to instructions to add the carried always to the first number in the next column.

During the instructional periods he was constantly checked on that habit by requesting him to do certain examples aloud.

To overcome the habit of grouping numbers, the procedure of adding the numbers in a successive order was illustrated. Also the possibilities of errors arising by such a procedure as he employed was explained. During the instructional periods he was encouraged to add the numbers in the successive order. Occasionally he was requested to work alone for purposes of checking.

Subtraction

Flash cards were employed to overcome the errors in primary subtraction.

To overcome subtraction habit 1, it was explained that zero in each case was not necessary. Following this, practice exercises were assigned and these were carefully watched for the reappearance of this habit.

Whenever it happened, he was requested to make corrections and reminded again that the zero is not essential in each case.

To overcome the habit of counting he studied the facts so that he could respond to them automatically.

To overcome subtraction habits 2 and 3, the method of subtracting even examples that caused him difficulty were illustrated. Following this,

he was requested to do similar examples orally. Later practice exercises were assigned for drill purposes.

Adding instead of subtracting might have been due to carelessness. No drill was given for purposes of overcoming this difficulty, however, when he added instead of subtracting during the instructional periods, the mistake was called to his attention always. In such cases he was requested to make the necessary correction.

Final Test Results. The corrective treatment of this Case is not complete. Table X on the following page shows the results of the tests that were administered.

	Date	Test	Score	Time
Addition	2/24/25	20	85	19:35
	2/25/25	25 (check form)	86	18:45
Subtraction	3/5/25	20	100	6:25

he was requested to do similar examples orally. Later primitive examples

were assigned for drill purposes.

Adding instead of subtracting might have been due to carelessness.

No drill was given for purposes of overcoming this difficulty, however,

when he added instead of subtracting during the instructional periods.

The mistake was called to his attention always. In such cases he was re-

quested to make the necessary correction.

Final Test Results. The corrective treatment of this case is not complete.

Table A on the following page shows the results of the tests that were ad-

ministered.

Summary of Case V. Donald was presented in the work and seemed capable of understanding it, still he calculated very poorly. His attendance was very irregular and uncertain. His scientific background was slight and he offered excuses for not doing them.

Table I shows the results of the three final tests that were administered. The other tests were not administered since remedial instruction could not be given.

TABLE X

Final Results of Case V on the Wilson Inventory
and Diagnostic Tests

Process	Date	Test	Score	Time
Addition	3/14/35	3C	99	19:55
	2/28/35	3P (short form)	96	18:42
Subtraction	3/5 /35	4A	100	3:28

Final Results of Case V on the Wilson Inventory
and Magnetic Tests

Process	Date	Test	Score	Time
Addition	2/14/33	35	33	12:33
	2/23/33	37 (short form)	35	12:45
Subtraction	2/2/33	34	100	3:28

Summary of Case V. Dominic was interested in the work and seemed capable of understanding it, still he cooperated very poorly. His attendance was very irregular and uncertain. His outside assignments were seldom completed and he offered excuses for not doing them.

Table X shows the results of the three final tests that were administered. The other tests were not administered since remedial instruction could not be continued because Dominic failed to report.

There were about twenty-four, forty-five minute periods spent on remedial instruction among addition and subtraction approximately as follows: addition 18 periods; and subtraction 6 periods.

Summary of Case V. Dominic was interested in the work and seemed capable of understanding it, still he cooperated very poorly. His attendance was very irregular and uncertain. His outside assignments were seldom completed and he offered excuses for not doing them.

Table X shows the results of the three final tests that were administered. The other tests were not administered since remedial instruction could not be continued because Dominic failed to report. There were about twenty-four, forty-five minute periods spent on remedial instruction among addition and subtraction approximately as follows: addition 18 periods; and subtraction 6 periods.

Summary

The final summary of this study is presented in a form of a table showing the evidence regarding the effects of the remedial instruction.

At the time this special study began Case I was in the sixth grade and he was 14 years and 2 months old. His IQ was found to be 85. He received about thirty, forty-five minute teaching periods on corrective work. They were divided among the processes approximately as follows: addition 7 periods; subtraction 5 periods; multiplication 6 periods; and division 12 periods. Final results show gains in every test and entirely satisfactory results in tests 3A, 3B, 4A, 4B, 5A, 5P and 6A. Tests 4P and 6P₂ show significant gains in score and time*respectively.

Case II with an IQ of 80 was 13 years and 7 months old and in the sixth grade at the time this study began. He received about forty-eight, forty-five minute teaching periods on corrective work. They were divided among the processes approximately as follows: addition 12 periods; subtraction 10 periods; multiplication 11 periods; and division 15 periods. Final results results produced perfect scores in tests 4A, 4B, 5A and 6A. Significant gain in time or score was made in tests 3C, 3P, 4P, 5P and 6P₂.

At the beginning of this study Case III was 10 years and 9 months old and in the fifth grade. His IQ was found to be 99. He received about fifty-three, twenty minute teaching periods on corrective work. They were divided among the processes approximately as follows: addition 22 periods; subtraction 12 periods; multiplication 11 periods; and division 8 periods. Final results produced perfect scores in tests 3A, 3B, 4A and 4B. Significant gain in score or time is reported in tests 3C, 3P, 5A, 5P and 6P₂.

*For method of transmuting recorded gains in time into grade gains see Appendix A.

When the study of Case IV began, he was 10 years and 8 months old and in the fifth grade. His IQ was found to be 90. He received about forty-five, twenty minute teaching periods on corrective work. They were divided among the processes approximately as follows: addition 14 periods; subtraction 12 periods; multiplication 11 periods; and division 8 periods. Final results produced perfect scores in tests 3A and 4P. Significant gain in score or time is reported in tests 5P and 6P₂.

Case V with an IQ of 96 was in the sixth grade and 11 years and 8 months old at the beginning of the study. In this Case there were about twenty-four, forty-five minute teaching periods devoted to corrective work among addition and subtraction approximately as follows; addition 18 periods; and subtraction 6 periods. Final results produced a perfect score in test 4A. A significant gain in time was reported in test 3C.

Limited time did not permit realization of the goal of perfect scores in all tests in every Case in a reasonably short time.

The most significant change in score reported was in Case III. On the initial attempt of test 5P, he received a score of twenty-four which indicates inaccuracy in work. At the end of the remedial instruction, he obtained a score of ninety-two on the same test. The final result indicates a gain in score and also increase in accuracy. In this same Case, the most significant reduction in time is reported. On the first attempt of test 3C, he spent 57 minutes and 2 seconds to complete it. After remedial instruction, he completed the same test in 24 minutes and 50 seconds which shows a reduction of 32 minutes and 12 seconds over the initial time. This reduction indicates that more efficient habits of work were employed. Complete summary of the results of the remedial instruction of the five case studies is shown in tabular form in Table XI which follows.

When the study of Case IV began, he was 10 years and 8 months old and in the fifth grade. His IQ was found to be 90. He received about forty-five, twenty minute teaching periods on corrective work. They were divided among the processes approximately as follows: addition 12 periods; subtraction 12 periods; multiplication 11 periods; and division 3 periods. Final results produced perfect scores in tests 3A and 3B. Significant gain in score or time is reported in tests 3C and 3D.

Case V with an IQ of 95 was in the fifth grade and 11 years and 8 months old at the beginning of the study. In this case there were about twenty-four, forty-five minute teaching periods devoted to corrective work among addition and subtraction approximately as follows: addition 12 periods and subtraction 6 periods. Final results produced a perfect score in test 3A. A significant gain in time was reported in test 3C.

Limited time did not permit realization of the goal of perfect scores in all tests in every case in a reasonably short time.

The most significant change in score reported was in Case III. On the initial attempt of test 3B, he received a score of twenty-four which indicated inaccuracy in work. At the end of the remedial instruction, he obtained a score of ninety-two on the same test. The final result indicates a gain in score and also increase in accuracy. In this same case, the most significant reduction in time is reported. On the first attempt of test 3B he spent 57 minutes and 2 seconds to complete it. After remedial instruction, he completed the same test in 24 minutes and 30 seconds which shows a reduction of 33 minutes and 12 seconds over the initial time. This reduction indicates that more efficient habits of work were employed. Complete summary of the results of the remedial instruction of the five cases studied is shown in tabular form in Table A1 which follows.

TABLE XI

Results of Remedial Instruction

Pupil	IQ	Age	Grade	Process	Teach. Periods	Minutes per Period	Initial Results			Final Results		
							Test	Score	Time	Test	Score	Time
Case I	85	14-2	6	Add.	7	45	3A	100	3A	100	3:40 **
							3B	99	3B	100	5:40 **
							3C	96+	3C	98+	17:45
							3P (short)	84	3P (short)	88	9:50
Case II	80	13-7	6	Sub.	5	45	4A	98	4:00	4A	100	3:55 **
							4B	99	10:00	4B	100	5:30 **
							4P (long)	78	39:30	4P (short)	96	6:45 *
							5A	95	8:00	5A	100	3:40 **
				Mult.	6	45	5P (long)	78	44:20	5P (short)	100	13:00 **
							6A	98	7:00	6A	100	3:27 **
				Div.	12	45	6P ₂	..	31:00	6P ₂	92	42:10 *
							3A	100	3:10			
				Sub.	10	45	3B	94	5:48			
							3C	95+	25:00	3C	98	16:10 *
							3P (short)	76	10:15	3P (short)	96	9:00 *
							4A	100	3:20	4A	100	3:00 **
				Mult.	11	45	4B	99	7:00	4B	100	6:41 **
							4P (long)	59	25:10	4P (short)	92	4:47 *
							5A	97	3:40	5A	100	2:40 **
							5P (long)	66	5P (short)	88	8:45 *
				Div.	15	45	6A	94	18:10	6A	100	5:45 **
							6P ₂	..	55:00	6P ₂	90	38:40 *

Table XI is read in the following manner: Case I has an IQ of 85. At the beginning of the study his age was 14 years and 2 months and he was in the sixth grade. Seven, forty-five minute teaching periods were devoted to corrective work in addition. On the initial test 3A, he obtained a score of 100. The final

score on test 3A was 100 and the time taken to complete the test was 3 minutes and 40 seconds. The data for the other cases are read likewise. Single asterisks indicate significant gain in score or time. Double asterisks indicate perfect scores.

TABLE XI (cont.)
Results of Remedial Instruction

Pupil	IQ	Age	Grade	Process	Teach. Periods	Minutes per Period	Initial Results		Final Results			
							Test	Score	Time	Test	Score	Time
Case III	99	10-9	5	Add.	22	20	3A	99	9:30	3A	100	4:00 **
							3B	100	13:32	3B	100	5:15 **
							3C	98	57:02	3C	99	24:50 *
							3P (short)	84	51:35	3P (short)	88	11:20 *
				Sub.	12	20	4A	100	8:00	4A	100	3:22 **
							4B	100	17:31	4B	100	5:20 **
							4P (long)	85	40:15	4P (short)	96	7:46
							5A	98	29:30	5A	99	8:06 *
				Mult.	11	20	5P (short)	24	28:00	5P (short)	92	24:40 *
							6A	96	10:53			
							6P ₂	55	43:28	6P ₂	76	25:15 *
Case IV	90	10-8	5	Add.	14	20	3A	100	3:10	3A	100	3:55 **
							3B	100	3:55	3B	99	4:01
							3P (short)	88	10:30	3P (short)	92	10:02
							4P (short)	96	14:02	4P (short)	100	8:42 **
				Sub.	12	20	5P (short)	72	29:06	5P (short)	88	17:02 *
							6A	91	19:06			
							6P ₂	55	41:17	6P ₂	85	31:04 *

TABLE XI (cont.)

Results of Remedial Instruction

Pupil	Iq	Age	Grade	Process	Teach. Periods	Minutes per Period	Initial Results			Final Results		
							Test	Score	Time	Test	Score	Time
Case V	96	11-8	6	Add.	18	45	3A	99	2:57			
							3B	96	6:06			
							3C	99	25:30	3C	99	19:55
							3P (long)	89	33:00	3P (short)	96	18:42
				Sub.	6	45	4P (long)	86	29:45	4A	100	3:28 **
				Mult.	0		5P (long)	53	57:20			
				Div.	0		6P ₂			

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Appendix A

Method of Transmuting the Data in the Wilson-Peterson Tests
into Grade Values

The reader can transform the scores in these or previous tests SP_1 , SP_2 , SP_3 and SP_4 of this study into grade values by using the following formula. A table of values is given on the back of the test to help in the calculation. The results of the transformation are reported in the second column of the following page. These results are as reported in the original work unless otherwise noted. The original work was based on the traditional basis. They were also transformed into the new basis by using the following formula:

Appendix A

Grade = $100 \times \frac{SP - 10}{90}$

The approximate grade in years is shown as follows: on the basis of the test scores of 11 and 12 years old, the scores of 13 and 14 years old are approximately 13 and 14 years old. The scores of 15 and 16 years old are approximately 15 and 16 years old. The scores of 17 and 18 years old are approximately 17 and 18 years old. The scores of 19 and 20 years old are approximately 19 and 20 years old. The scores of 21 and 22 years old are approximately 21 and 22 years old. The scores of 23 and 24 years old are approximately 23 and 24 years old. The scores of 25 and 26 years old are approximately 25 and 26 years old. The scores of 27 and 28 years old are approximately 27 and 28 years old. The scores of 29 and 30 years old are approximately 29 and 30 years old. The scores of 31 and 32 years old are approximately 31 and 32 years old. The scores of 33 and 34 years old are approximately 33 and 34 years old. The scores of 35 and 36 years old are approximately 35 and 36 years old. The scores of 37 and 38 years old are approximately 37 and 38 years old. The scores of 39 and 40 years old are approximately 39 and 40 years old. The scores of 41 and 42 years old are approximately 41 and 42 years old. The scores of 43 and 44 years old are approximately 43 and 44 years old. The scores of 45 and 46 years old are approximately 45 and 46 years old. The scores of 47 and 48 years old are approximately 47 and 48 years old. The scores of 49 and 50 years old are approximately 49 and 50 years old. The scores of 51 and 52 years old are approximately 51 and 52 years old. The scores of 53 and 54 years old are approximately 53 and 54 years old. The scores of 55 and 56 years old are approximately 55 and 56 years old. The scores of 57 and 58 years old are approximately 57 and 58 years old. The scores of 59 and 60 years old are approximately 59 and 60 years old. The scores of 61 and 62 years old are approximately 61 and 62 years old. The scores of 63 and 64 years old are approximately 63 and 64 years old. The scores of 65 and 66 years old are approximately 65 and 66 years old. The scores of 67 and 68 years old are approximately 67 and 68 years old. The scores of 69 and 70 years old are approximately 69 and 70 years old. The scores of 71 and 72 years old are approximately 71 and 72 years old. The scores of 73 and 74 years old are approximately 73 and 74 years old. The scores of 75 and 76 years old are approximately 75 and 76 years old. The scores of 77 and 78 years old are approximately 77 and 78 years old. The scores of 79 and 80 years old are approximately 79 and 80 years old. The scores of 81 and 82 years old are approximately 81 and 82 years old. The scores of 83 and 84 years old are approximately 83 and 84 years old. The scores of 85 and 86 years old are approximately 85 and 86 years old. The scores of 87 and 88 years old are approximately 87 and 88 years old. The scores of 89 and 90 years old are approximately 89 and 90 years old. The scores of 91 and 92 years old are approximately 91 and 92 years old. The scores of 93 and 94 years old are approximately 93 and 94 years old. The scores of 95 and 96 years old are approximately 95 and 96 years old. The scores of 97 and 98 years old are approximately 97 and 98 years old. The scores of 99 and 100 years old are approximately 99 and 100 years old.

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Appendix A

Method of Transmuting Time Gains in the Wilson Process Tests
into Grade Gains

The reader can transmute the change in time on process tests 3P, 4P, 5P and 6P₂ of this study that show gain in scores to approximate gain in years on the basis of time as set up in the manual of these tests. A copy is attached on the following page. Such results as are recorded in the manual were secured from children taught on the traditional basis. They show average performance of children in different grades at the time, (about 1927-28).

The approximate gain in years is secured as follows; on the initial attempt Case II took 25 minutes and 10 seconds to complete test 4P. According to the median results in the manual, Case II would, therefore, fall somewhere between grades 5 and 6. Following remedial instruction he completed the short form which is one-fourth as long as the long form in 4 minutes and 47 seconds. For purposes of comparison the time taken to complete the short form must be multiplied by 4. The new time, 19 minutes and 8 seconds, indicates that after remedial instruction he was doing better than the median for the eighth grade. This shows that the remedial instruction produced better than a two year gain. The approximate gain in years for the other process tests can also be computed likewise.

Appendix A

Method of Transmuting Time Gains in the Wilson Process Tests
into Grade Gains

The reader can transmute the change in time on process tests 45, 46, 47 and 48 of this study that show gain in scores to approximate gain in years on the basis of time as set up in the manual of these tests. A copy is attached on the following page. Such results as are recorded in the manual were secured from children taught on the traditional basis. They show average performance of children in different grades at the time.

(about 1937-38).

The approximate gain in years is secured as follows; on the initial strategy Case II took 35 minutes and 10 seconds to complete test 45. According to the median results in the manual, Case II would, therefore, fail somewhere between grades 5 and 6. Following remedial instruction he completed the short form which is one-fourth as long as the long form in 4 minutes and 47 seconds. For purposes of comparison the time taken to complete the short form must be multiplied by 4. The new time, 19 minutes and 48 seconds, indicates that after remedial instruction he was doing better than the median for the eighth grade. This shows that the remedial instruction produced better than a two year gain. The approximate gain in years for the other process tests can also be computed likewise.

The Wilson Process Inventory and Diagnostic Tests in Arithmetic

A Brief Manual of Instructions

By G. M. WILSON

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